

Contents

Chapter 1	Fundamental Concepts	1
1	Introduction	1
2	A Simple Physical Model	2
3	A Classical Approach	2
4	The Invariant Imbedding Approach	6
5	Some Comments, Criticisms, and Questions	9
6	A Minor Variant of the Model of Section 2	9
7	A Major Variant of the Model of Section 2	10
8	The Classical Approach Extended	11
9	The Invariant Imbedding Approach Extended	13
10	Some Comments on Possible Uses of the Reflection and Transmission Functions	15
11	Summary	16
	Problems	17
	References	20
Chapter 2	Additional Illustrations of the Invariant Imbedding Method	22
1	Introduction	22
2	A Non-Linear Problem	23
3	A Generalization of the Model	24
4	Invariant Imbedding Formulation of the Model in Section 3	25
5	The Linear Problem Revisited	27
6	A Perturbation Approach	28
7	Some Remarks and Comments	31
8	The Riccati Transformation Method	32
9	Summary	35
	Problems	35
	References	38
Chapter 3	Functional Equations and Related Matters	39
1	Introduction	39
2	A Basic Problem	39
3	The Basic Functional Equations	40
4	Some Applications of the Results of Section 3	41
5	Differential Equations Via Functional Equations	45
6	Summary	47

Problems	47
References	53

Chapter 4 Existence, Uniqueness, and Conservation Relations 54

1	Introduction	54
2	The "Physics" of the Conservative Case and Its Generalizations	55
3	Another Derivation of the Reflection Function	56
4	Some Conservation Relations	57
5	Proof of Existence in the Conservative Case	59
6	The Nonconservative Case: The Dissipation Function	60
7	The Existence Proof	63
8	Summary	64
	Problems	65
	References	66

Chapter 5 Random Walk 67

1	Introduction	67
2	A One-Dimensional Random Walk Process	67
3	A Classical Formulation	68
4	An Invariant Imbedding Formulation	70
5	Some Remarks Concerning Section 4	71
6	Sketch of Another Approach	72
7	Expected Sojourn	73
8	A "Many-State" Case—Invariant Imbedding Approach	74
9	Time Dependent Processes—Classical Approach	76
10	Time-dependent Processes—Invariant Imbedding Approach	77
11	A Multistep Process—Classical Approach	78
12	A Multistep Process—Invariant Imbedding Approach	79
13	Some Remarks on an Extension to a Continuous Case	81
14	Some Remarks About Random Walk in Two Dimensions	82
15	Summary	83
	Problems	83
	References	87

Chapter 6 Wave Propagation 88

1	Introduction	88
2	The Concept of a Plane Wave	89
3	A Two Medium Problem	89
4	A Multimedium Problem	90
5	Resolution of the Multimedium Problem by "Wavelet Counting"	91
6	A Continuous Medium Problem	94
7	An Analytical Approach to the Continuous Medium Problem	97

8	The W. K. B Method	98
9	The Bremmer Series	100
10	Another Imbedding	103
11	Summary	105
	Problems	105
	References	107
Chapter 7 Time-Dependent Problems		
		108
1	Introduction	108
2	A Time-Dependent Transport Problem—Particle-Counting Approach	109
3	Time-Dependent Transport by Transform Techniques	114
4	A Critique of the Foregoing	116
5	Time-Dependent Input	118
6	The Time-Dependent Wave Equation	120
7	The Diffusion Equation	121
8	Some Comments on the Previous Section	124
9	A Critique of Sections 7 and 8	125
10	Another Diffusion Problem	126
11	A Final Diffusion Problem	127
12	Summary	129
	Problems	130
	References	132
Chapter 8 The Calculation of Eigenvalues for Sturm-Liouville Type Systems		
		133
1	Introduction	133
2	Eigenlengths for Transport-like Equations in One Dimension	134
3	The Calculation of Eigenlengths	134
4	Some Generalizations	137
5	Results for Sturm-Liouville Systems	138
6	Connection with the Prüfer Transformation	140
7	Some Numerical Examples	142
8	Summary	144
	Problems	145
	References	146
Chapter 9 Schrödinger-Like Equations		
		147
1	Introduction	147
2	Formulation of the Phase Shift Problem	148
3	A Representation of the Solution for Large t	149
4	Partial Differential Equations for a and ψ	151
5	Solution of the Partial Differential Equations for a and ψ	153
6	Remarks on the Phase Shift Problem	154

7	Formulation of the Eigenvalue Problem	155
8	A Partial Differential Equation for \bar{b} and Its "Solution"	158
9	Resolution of the Difficulties	161
10	Some Numerical Examples	164
11	Some Remarks on the Eigenvalue Problem	165
12	Summary	166
	Problems	167
	References	168
Chapter 10 Applications to Equations with Periodic Coefficients		169
1	Introduction	169
2	Statement of the Problem	170
3	The Differential Equations of Invariant Imbedding Over One Period	171
4	Difference Equations Over an Integral Number of Periods	171
5	Difference Equations Over a Nonintegral Number of Periods	176
6	The "Backwards" Equations	177
7	Some Numerical Results	178
8	The Method of Doubling	180
9	Trigonometry Revisited	181
10	Summary	182
	Problems	183
	References	185
Chapter 11 Transport Theory and Radiative Transfer		186
1	Introduction	186
2	The Linearized Boltzmann Equation	187
3	Some Remarks on Sections 1 and 2	190
4	Boundary and Initial Conditions	192
5	The Special Case of Slab Geometry and One Speed	193
6	The Time-Independent Slab Problem via Invariant Imbedding—The Perturbation Approach	196
7	The Time-Independent Slab Problem Via Invariant Imbedding—The Riccati Transformation	202
8	A Return to the Case of the Semi-Infinite Half-Space	206
9	Invariant Imbedding as a Computational Device for Transport Problems in a Slab	208
10	Transport Theory in Other Geometries	209
11	Time-Dependent Transport in a Slab Geometry	210
12	Summary	214
	Problems	215
	References	217

CONTENTS	xvii
Chapter 12 Integral Equations	219
1 Introduction	219
2 An Integral Equation for Transport in a Slab	220
3 A Pseudo-Transport Problem and Its Associated Integral Equation	223
4 Representations for ϕ and n	226
5 Derivation of the Principal Results	227
6 A Special Case	232
7 A Numerical Example and Some Remarks About Eigenvalues	234
8 Further Remarks About the Foregoing	238
9 A Completely Different Approach	239
10 Summary	242
Problems	242
References	244
Author Index	245
Subject Index	247