

CONTENTS

INTRODUCTION 1

1. The inverse of a nonsingular matrix 1
2. Generalized inverses of matrices 1
3. Illustration: Solvability of linear systems 2
4. Diversity of generalized inverses 3
5. Preparation expected of the reader 4
6. Historical note 4
7. Remarks on notation 5

1. EXISTENCE AND CONSTRUCTION OF GENERALIZED INVERSES 7

1. The Penrose equations 7
2. Existence and construction of $\{1\}$ -inverses 8
3. Properties of $\{1\}$ -inverses 11
4. Bases for the range and null space of a matrix 14
5. Existence and construction of $\{1,2\}$ -inverses 18
6. Existence and construction of $\{1,2,3\}$ -, $\{1,2,4\}$ -, and $\{1,2,3,4\}$ -inverses 20
7. Full-rank factorizations 22
8. Explicit formula for A^\dagger 23
9. Construction of $\{2\}$ -inverses of prescribed rank 25
10. An application of $\{2\}$ -inverses in iterative methods for solving nonlinear equations 27

2. LINEAR SYSTEMS AND CHARACTERIZATION OF GENERALIZED INVERSES 39

1. Solution of linear systems 39
2. Characterization of $A\{1,3\}$ and $A\{1,4\}$ 43
3. Characterization of $A\{2\}$, $A\{1,2\}$ and other subsets of $A\{2\}$ 46
4. Idempotent matrices and projectors 48
5. Generalized inverses with prescribed range and null space 58

6. Orthogonal projections and orthogonal projectors 63
 7. Efficient characterization of classes of generalized inverses 77
 8. Restricted generalized inverses 81
 9. The Bott–Duffin inverse 86
 10. An application of $\{1\}$ -inverses in interval linear programming 90
 11. A $\{1,2\}$ -inverse for the integral solution of linear equations 93
 12. An application of the Bott–Duffin inverse to electrical networks 96
3. MINIMAL PROPERTIES OF GENERALIZED INVERSES 103
1. Least-squares solutions of inconsistent linear systems 103
 2. Solutions of minimum norm 113
 3. Weighted generalized inverses 121
 4. Essentially strictly convex norms and the associated projectors and generalized inverses 128
 5. An extremal property of the Bott–Duffin inverse with application to electrical networks 155
4. SPECTRAL GENERALIZED INVERSES 159
1. Introduction 159
 2. Spectral properties of a nonsingular matrix 160
 3. Spectral inverse of a diagonable matrix 161
 4. The group inverse 162
 5. Spectral properties of the group inverse 166
 6. The Drazin pseudoinverse. Index of a square matrix 169
 7. Spectral properties of the Drazin pseudoinverse 175
 8. Index 1-nilpotent decomposition of a square matrix 175
 9. Quasi-commuting inverse 178
 10. Other spectral generalized inverses 179
5. GENERALIZED INVERSES OF PARTITIONED MATRICES 186
1. Introduction 186
 2. Partitioned matrices and linear equations 187
 3. Intersection of manifolds 197
 4. Common solutions of linear equations and generalized inverses of partitioned matrices 208
 5. Greville's method and related results 219
 6. Generalized inverses of bordered matrices 228

6. A SPECTRAL THEORY FOR RECTANGULAR MATRICES 233
 1. Introduction 233
 2. The UDV^* decomposition 242
 3. Partial isometries and the polar decomposition theorem 252
 4. A spectral theory for rectangular matrices 270

7. COMPUTATIONAL ASPECTS OF GENERALIZED INVERSES 283
 1. Introduction 283
 2. Computation of unrestricted $\{1\}$ -inverses and $\{1,2\}$ -inverses 284
 3. Computation of unrestricted $\{1,3\}$ -inverses 285
 4. Computation of $\{2\}$ -inverses with prescribed range and null space 287
 5. Iterative methods for computing A^\dagger 290

8. GENERALIZED INVERSES OF LINEAR OPERATORS BETWEEN HILBERT SPACES 306
 1. Introduction 306
 2. Hilbert spaces and operators: Preliminaries and notation 307
 3. Generalized inverses of linear operators between Hilbert spaces 314
 4. Minimal properties of generalized inverses 342
 5. Series and integral representations and iterative computation of generalized inverses 351

BIBLIOGRAPHY 359

GLOSSARY OF SYMBOLS 383

AUTHOR INDEX 387

SUBJECT INDEX 393