CONTENTS

1. Introduction

General remarks 1

Dual sequence spaces 2

2. Notation and terminology 3

Standard sequence spaces 3

Group norms 5

Generalized Köthe-Toeplitz duals; matrix classes 8

Theorems of Kojima-Schur, Toeplitz, Schur 10

Theorem of Crone 15

Strong summability 17

Abel, Cesàro, Nörlund summability 18

3. Generalized Köthe-Toeplitz duals 19

$c_o^{\beta}(X)$ 19

$c^{\beta}(X)$ 20

$\ell_{\infty}^{\beta}(X)$ 21

$c_o^{\alpha}(X)$ 23

$\ell_1(B(X,Y)) \subset \ell_{\infty}^{\beta}(X)$ 24

$\ell_p^{\beta}(X)$, $0 < p \leq 1$ 26

$\ell_p^{\beta}(X)$, $1 < p < \infty$ 27

$\ell_p^{\alpha}(X)$, $1 < p < \infty$ 29

$(A_k) \in \ell_{\infty}^{\beta}(X)$ implies ΣA_k unconditionally convergent in the uniform operator topology 30

$w_p^{\beta}(X)$, $0 < p < 1$ 31

$w_p^{\alpha}(X)$, $0 < p < 1$, and $w_p^{\beta}(X)$, $1 \leq p < \infty$ 33

4. Characterization of matrix classes 36

Theorem of Lorentz-Macphail 36

Theorem of Robinson on $(c(X), c(Y))$ 38

Theorem of Toeplitz for operators 40

Theorem of Steinhaus for operators 41

Sum preserving series to sequence maps 43

Theorem of Schur for operators 46

$pA \leq Mp$ on $\ell_\infty(X)$ 51

$(\ell_\infty(X), c_o(Y))$, and $(\ell_1(X), \ell_p(Y))$, $1 \leq p < \infty$ 53

$(\ell_1(X), \ell_\infty(Y))$ 55

The space $D = \bigoplus_2 H_i$ 56

Theorem of Crone for operators 61

$(w_p(X), c(Y))$; Theorem of Kuttner for operators 63

5. Tauberian theorems 65

Theorems of Tauber and Hardy 65

Theorem of Littlewood; the Tauberian condition $\sup n||a_n|| < \infty$ in a Banach space 66

Theorem of Northcott 67

A Tauberian theorem for regular additive summability methods 72

A regular method for which $na_n \to 0$ is a Tauberian condition but $na_n \to m$ is not 74

A generalization of Tauber's second theorem 78

A Tauberian theorem in ℓ_p $(0 < p < 1)$ 80

6. Consistency theorems 84

E-consistency of operator matrices 84

Consistency is not an equivalence relation 85

Classicial bounded consistency theorem; Petersen's approach 86

Mazur-Orlicz bounded consistency theorem 91

Zeller's approach 92

Theorem of Copping on conull matrices 93

Form of $c^*(X)$ and $c_0^*(X)$ 94

Two-norm spaces, γ-convergence, completeness, density, continuity; lower semicontinuity; the Σ_1 postulate 96

Finite sequences γ-dense in $\ell_\infty(X) \cap (A)_0$ 99

The Alexiewicz-Orlicz bounded consistency theorem for operators 105

7. Operator Nörlund means 106

(N,q,X) summability 106

Means with equal summability fields 107

A theorem of consistency 109

Exact conditions for $c(X) = (N,q,X)$ 111

A consistency theorem for regular operator Nörlund means 113

An open question on regular complex means 114

Consistency of regular real Nörlund means 115

Bibliography 117

List of symbols 122