

# Contents

## Volume 2

|  |            |
|--|------------|
| <b>CHAPTER 5. APPLICATION OF THE THEORY OF EXPANSIONS<br/>TO HARMONIC ANALYSIS</b> | <b>1</b>   |
| <b>§ 1. <i>Some Criteria of the Selfadjointness of Operators</i></b>               | <b>4</b>   |
| 1.1. Schrödinger and hyperbolic criteria of selfadjointness                        | 4          |
| 1.2. Some generalizations of the hyperbolic criterion                              | 13         |
| 1.3. Quasianalytical criteria of selfadjointness                                   | 21         |
| 1.4. Parabolic criteria of selfadjointness   | 28         |
| 1.5. Selfadjointness of perturbations of an operator by a potential                | 31         |
| 1.6. Another evolutionary criterion of selfadjointness                             | 39         |
| 1.7. Approximation theorem   | 39         |
| 1.8. Verification of the commutativity of selfadjoint operators                    | 44         |
| <b>§ 2. <i>Generalized Power Moment Problem</i></b>                                | <b>51</b>  |
| 2.1. Generalized symmetric power moment problem                                    | 52         |
| 2.2. Examples  | 73         |
| 2.3. Two more proofs of the theorem on representation                              | 82         |
| 2.4. Generalized power moment problem  | 85         |
| 2.5. A family of shift operators and the corresponding Fourier transform           | 94         |
| 2.6. Some additional remarks   | 100        |
| <b>§ 3. <i>Positive Definite Functions of Infinitely Many Variables</i></b>        | <b>102</b> |
| 3.1. Some auxiliary facts  | 102        |
| 3.2. Positive definite functions and their spectral representation                 | 106        |
| 3.3. Positive definite functions on a layer  | 121        |
| <b>§ 4. <i>Positive Definite Functions on a Hilbert Space</i></b>                  | <b>125</b> |
| 4.1. Positive definite functions on a layer in the Hilbert space                   | 125        |

|             |   |            |
|-------------|---|------------|
| 4.2.        | Extension of a positive definite function from a layer in the Hilbert space to the whole of the space. Connection with the moment problem | 137        |
| 4.3.        | Other formulations of the results concerning positive definite functions on Hilbert spaces  | 146        |
| 4.4.        | Connection between unitary representations of a Hilbert space and positive definite functions on this space                               | 149        |
| 4.5.        | Some additional remarks   | 157        |
| <b>§ 5.</b> | <b><i>General Scheme of the Representation of Positive Definite Kernels in Terms of Elementary Ones</i></b>                               | <b>160</b> |
| 5.1.        | Hilbert space constructed for a given positive definite kernel and its rigging  | 160        |
| 5.2.        | Operators acting in the space generated by a positive definite kernel   | 165        |
| 5.3.        | Spectral representation of positive definite kernels in terms of elementary ones  | 170        |
| 5.4.        | The case of smooth positive definite kernels  | 175        |

## **CHAPTER 6. INFINITE-DIMENSIONAL ELLIPTIC DIFFERENTIAL OPERATORS OF THE SECOND ORDER** **181**

|             |  |            |
|-------------|--|------------|
| <b>§ 1.</b> | <b><i>Second Quantization Operators in the Schrödinger Representation</i></b>            | <b>184</b> |
| 1.1.        | The second quantization operators  | 184        |
| 1.2.        | Mehler's formula in the infinite case  | 188        |
| 1.3.        | Definition of the second quantization operators by using a differential expression       | 195        |
| 1.4.        | Properties of the second quantization operators  | 202        |
| 1.5.        | Functional integrals corresponding to the second quantization operators                  | 209        |
| <b>§ 2.</b> | <b><i>Perturbations of the Second Quantization Operators by Potentials</i></b>           | <b>224</b> |
| 2.1.        | Kato's inequality and its consequences   | 225        |
| 2.2.        | The Feynman–Kac formula  | 231        |
| 2.3.        | Properties of perturbed operators  | 238        |
| 2.4.        | More singular potential perturbations. Interactions in constructive quantum field theory | 245        |

|   |            |
|---|------------|
| <b>§ 3. <i>Dirichlet Forms and Differential Operators Corresponding to These Forms</i></b>  | 248        |
| 3.1. Dirichlet forms of probability measures and Dirichlet operators  | 249        |
| 3.2. Renormalization of potential perturbations of the second quantization operators by use of the Dirichlet forms                        | 255        |
| 3.3. Dirichlet operators in the case of measures defined on a Hilbert space   | 263        |
| <b>§ 4. <i>Infinite-Dimensional Differential Equations</i></b>  | 276        |
| 4.1. Coercivity inequalities and their consequences   | 276        |
| 4.2. Hyperbolic equations corresponding to Dirichlet operators  | 281        |
| 4.3. Potential perturbations of Dirichlet operators   | 293        |
| <b>CHAPTER 7. INFINITE-DIMENSIONAL DIFFERENTIAL OPERATORS IN THE MODELS OF QUANTUM STATISTICAL PHYSICS AND FIELD THEORY</b>               | <b>299</b> |
| <b>§ 1. <i>The General Scheme of Constructing Renormalized Operators</i></b>  | 302        |
| 1.1. Operator realization of formal Hamiltonians  | 303        |
| 1.2. The functional integral for a renormalized operator  | 310        |
| <b>§ 2. <i>Several Field Theory Models</i></b>  | 314        |
| 2.1. Linear interaction (the Van Hove model)  | 315        |
| 2.2. Quadratic interaction  | 321        |
| <b>§ 3. <i>The Dirichlet Operators in Quantum Statistical Physics</i></b>   | 326        |
| 3.1. Construction of the dynamics of quantum lattice systems  | 326        |
| 3.2. Gaussian measures  | 333        |
| 3.3. Time evolution in harmonic systems   | 340        |
| 3.4. Translation invariant harmonic systems   | 344        |
| 3.5. Some remarks concerning the construction of time evolution in models with anharmonicity  | 347        |
| <b>§ 4. <i>Investigation of the Spectral Properties of Systems with Infinitely Many Particles by the Methods of Scattering Theory</i></b> | 349        |
| 4.1. Scattering problem for Dirichlet operators   | 350        |

|   |            |
|---|------------|
| 4.2. Perturbation of a measure in the Gaussian model and the wave operators corresponding to it | 357        |
| 4.3. Potential perturbations of harmonic systems  | 360        |
| 4.4. Scattering problem in the case of exactly solvable models                                  | 365        |
| <b>Bibliographical Notes</b>  | <b>371</b> |
| <b>References</b>   | <b>385</b> |
| <b>Subject Index of Volume 2</b>  | <b>425</b> |
| <b>List of Notations of Volume 2</b>  | <b>427</b> |
| <b>Contents of Volume 1</b>   | <b>429</b> |