

## CONTENTS

PREFACE	v
1 PRELIMINARIES: SET THEORY AND GENERAL TOPOLOGY	1
1.1 The Algebra of Sets	1
1.2 Partially Ordered Sets	4
1.3 Topology and Topological Spaces	6
1.4 Baire's Theorem	16
2 BANACH SPACES	19
2.1 Linear Spaces	19
2.2 Linear Independence	21
2.3 Sets in Linear Spaces	22
2.4 Classes of Spaces: Isomorphic Spaces, Quotient Spaces, and Complementary Spaces	25
2.5 Seminorms and Norms on Linear Spaces	27
2.6 Linear Topological Spaces	29
2.7 Banach Spaces	30
2.8 Linear Operators on Banach Spaces	32
2.9 Uniformly Convex and Rotund Banach Spaces: Some Generalizations	35
2.10 The Hahn-Banach Extension Theorem	39
2.11 Extension Theorems for Complex Banach Spaces	43
2.12 Three Basic Theorems of Linear Analysis	45
2.13 Convergence in Banach Spaces	56
2.14 The Adjoint of an Operator	63
2.15 The Spectrum of an Operator	65
2.16 The Local Spectrum of an Operator	70
2.17 Analytic Representation of the Dual of Some Banach Spaces	76
2.18 Measures of Noncompactness and Classes of Mappings on Banach Spaces	84

3	HILBERT SPACES	109
3.1	Inner Products on Linear Spaces	109
3.2	Orthonormal Bases and the Bessel Inequality	
3.3	Separable Hilbert Spaces: Gram-Schmidt Orthogonalization Method	119
3.4	Orthogonal Subspaces of a Hilbert Space	125
3.5	The Dual of a Hilbert Space	126
3.6	Classes of Bounded Linear Operators on Hilbert Spaces	129
4	BANACH ALGEBRAS	142
4.1	Definitions and Some Examples	142
4.2	The Spectrum of an Element in a Banach Algebra with Unit	147
4.3	Representation Theorems for Commutative Banach Algebras	151
4.4	Structure Theorems for Commutative Banach Algebras	154
4.5	Representation Theorems for Noncommutative Banach Algebras	167
5	SPECTRAL REPRESENTATION OF OPERATORS ON HILBERT SPACES	174
5.1	Semispectral and Spectral Families of Radon Measures	174
5.2	Measurability and Integrability with Respect to Spectral Families	179
5.3	A Representation Theorem for $L^\infty$	181
5.4	Spectral Decomposition of Some Classes of Operators	188
5.5	Some Remarks on the Spectral Mapping Theorem for Hermitian and Normal Operators	191
6	THE NUMERICAL RANGE	200
6.1	The Numerical Range for Bounded Linear Operators on Hilbert Spaces	200
6.2	The Numerical Range and the Spectrum	206
6.3	The Numerical Range and Its Closure	214
6.4	The Essential Numerical Range for Bounded Linear Operators on Hilbert Spaces	217
6.5	The Maximal Numerical Range of a Bounded Operator on a Hilbert Space	219
6.6	The Extreme Points of the Numerical Range for Hyponormal Operators and (WN) Operators	221
6.7	The Numerical Range and Some Classes of Operators	223
6.8	The Numerical Range and Tensor Products	224

6.9	The Numerical Range for Bounded Linear Operators on Banach Spaces	226
6.10	The Exponential Function on the Set of All Bounded Linear Operators on a Banach Space	233
6.11	The Numerical Radius, the Spectral Radius, and the Norm of a Bounded Linear Operator on a Banach Space	236
6.12	Hermitian and Normal Operators on Banach Spaces	238
6.13	Normal Operators on Banach Spaces	244
6.14	Classes of Elements in Banach Algebras with Unit: The Vidav-Palmer Theorem	246
6.15	Some Properties of Hermitian and Normal Elements of a Banach Algebra	255
6.16	The Numerical Radius and the Iterates of an Element	260
6.17	The Numerical Range of Elements of Locally $m$ -Convex Algebras	262
7	NONNORMAL CLASSES OF OPERATORS	265
7.1	Classes of Nonnormal Operators	266
7.2	Spectral Sets and Dilations of Operators	275
7.3	Operators with $G_1$ Property and Some Generalizations	286
7.4	Operators with Property $\text{Re } \sigma(T) = \sigma(\text{Re } T)$	297
7.5	The Class $\bar{R}_1$	302
7.6	Other Classes of Bounded Operators	307
8	CONDITIONS IMPLYING NORMALITY	310
8.1	Conditions Implying Hermitianity	310
8.2	Conditions Implying Unitarity	317
8.3	Conditions Implying Normality	321
9	SYMMETRIZABLE OPERATORS: GENERALIZATIONS AND APPLICATIONS	344
9.1	Symmetrizable Operators on Hilbert Spaces	344
9.2	Symmetrizable Elements in Banach Algebras	349
9.3	Inner Products on Banach Spaces: Symmetrizable Operators and Some Generalizations	352
9.4	Some Applications of Symmetrizable Operators and Quasi-Normalizable Operators	369
9.5	Further Results on Symmetrizable Operators on Hilbert Spaces	374

10	INVARIANT SUBSPACES AND SOME STRUCTURE THEOREMS	380
10.1	Invariant Subspaces: Some Existence Theorems	380
10.2	Reducing Invariant Subspaces	398
10.3	Some Structure Theorems	408
11	THE WEYL SPECTRUM OF AN OPERATOR	412
11.1	Preliminaries and Some General Results	412
11.2	Weyl's Theorem	420
11.3	Weyl's Theorem for Some Classes of Operators	426
11.4	The Weyl Spectrum of an Element in a von Neumann Algebra	433
11.5	The von Neumann Theorem	439
12	ANALYTIC AND QUASI-ANALYTIC VECTORS	444
12.0	Introduction	444
12.1	Self-Adjoint Operators	446
12.2	Classes of Vectors for an Operator	452
12.3	Analytic and Quasi-Analytic Vectors and Essentially Self-Adjoint Operators	455
12.4	Quasi-Analytic Vectors and Semigroups of Operators	463
12.5	Analytic and Quasi-Analytic Elements in Commutative Banach Algebras	466
13	SCHWARZ NORMS	468
13.1	Schwarz Norms	469
13.2	A New Class of Schwarz Norms	476
13.3	Schwarz Norms on Banach Spaces	478
14	MAXIMUM THEOREMS FOR OPERATOR-VALUED HOLOMORPHIC FUNCTIONS	483
14.1	Holomorphic Functions	483
14.2	Subharmonic Functions	485
14.3	Maximum Theorems for the Norm	498
14.4	Maximum Theorems for the Spectral Radius and for the Essential Spectral Radius	508
14.5	Maximum Theorems for Other Operator-Valued Holomorphic Functions	528
15	UNIFORM ERGODIC THEOREMS FOR SOME CLASSES OF OPERATORS	537
15.1	Classes of Operators	537
15.2	Applications to Markov Processes	545

Contents	xi
APPENDIX. C <sub>p</sub> CLASSES	549
REFERENCES	553
SYMBOL INDEX	573
SUBJECT INDEX	575