## **Contents**

Foreword to the English translation	
Foreword	xi
Introduction	1
CHAPTER 1. Spaces of Generalized Functions	9
§1. The concept of a space with negative norm	10
1. Definition of the negative space	10
2. The Hilbert property of the negative space	12
3. Coincidence of the negative space with the dual of the	
positive space	12
4. Construction of isometries between the spaces in the chain	13
5. Adjointness with respect to the zero space	14
6. Construction of a chain from a subspace of the positive space	16
7. Construction of a chain from the negative space	16
8. Construction of a chain from an operator	18
9. Weighted orthogonal sums of chains	18
10. Projective limits of Banach spaces	19
§2. Finite and infinite tensor products of Hilbert spaces	23
1. Tensor products of finitely many Hilbert spaces	23
2. Tensor products of finitely many operators	25
3. Tensor products of infinitely many Hilbert spaces	27
4. Imbeddings of tensor products	30
5. Tensor products of chains	33
6. Weighted tensor products of Hilbert spaces	36
7. Tensor products of infinitely many operators	40
8. The kernel theorem	43
9. The case of bilinear forms. Positive-definite kernels	49
10. Complete von Neumann tensor product of infinitely many	
Hilbert spaces	51
11. Triplets of spaces	57

iv CONTENTS

33. Spaces of functions of finitely many variables	60
1. Spaces of square-integrable functions with a weight	60
2. Sobolev spaces on a bounded domain	60
3. The delta function as an element of a negative Sobolev space	62
4. Quasinuclearity of imbeddings of Sobolev spaces	63
5. The kernel theorem in spaces of square-integrable functions	
on bounded domains	66
6. The spaces $W_2^l$	69
7. Sobolev spaces on an unbounded domain	70
8. Sobolev spaces with a weight	73
9. The Schwartz space $S(\mathbb{R}^N)$ as a projective limit of Sobolev	
spaces	75
10. The space $\mathcal{D}(\mathbb{R}^N)$ as a projective limit of Sobolev spaces	77
11. The kernel theorem in spaces of square-integrable functions	79
12. The chain constructed from a continuous positive-definite	
kernel	83
§4. Spaces of functions of infinitely many variables	86
1. The space of square-integrable functions with respect to a	
product measure and the rigging of it	86
2. The case of a Gaussian measure	87
3. Weighted infinite tensor products of nuclear spaces	92
4. Examples of infinite tensor products of nuclear spaces	95
5. The space $\mathcal{A}(\mathbb{R}^{\infty})$	98
6. The space $\mathcal{H}(\mathbb{R}^{\infty})$	107
CHAPTER 2. General Selfadjoint and Normal Operators	113
§1. A joint resolution of the identity	117
1. A general resolution of the identity	117
2. Products of finitely many resolutions of the identity	119
3. Construction of a joint resolution of the identity in the	110
general case	121
4. Topologization	122
5. Regularity of a joint resolution of the identity	124
6. The concept of the support of a measure and its properties	125
7. Families of multiplication operators	128
8. Construction of a measure on a larger space from a measure	
on a smaller space. Compactification	130
9. Construction of a measure on a smaller space from a measure	
on a larger space. Modification of a measure	132
10. Properness of a joint resolution of the identity	135
11. Spectral representation of a family of commuting normal	
operators	138

CONTENTS

§ <b>2</b> .	The spectral theorem	139
	1. Differentiation of an operator-valued measure with respect to	
	its trace	139
	2. Differentiation of a resolution of the identity. The spectral	
	measure	144
	3. Differentiation of a joint resolution of the identity	146
	4. The case of a nuclear rigging	147
	5. The concept of a joint eigenvector and the spectrum of a family of operators	148
	6. The spectral theorem. The case of at most countably many	
	operators	149
	7. The spectral theorem. The general case	151
	8. The spectral theorem. The case of at most countably many	
	unbounded operators	163
	9. Continuity and smoothness of eigenvalues	164
	10. Supplementary remarks	164
	11. The Fourier transformation. The direct integral of Hilbert	
	spaces	165
§ <b>3</b> .	Connections with random processes and commutative algebras of	
•	operators. Quantum processes	170
	1. The concept of a random process	170
	2. Construction of a family of commuting normal operators	
	from a random process	171
	3. Construction of a random process from a family of	
	commuting normal operators. Connection with commutative	
	algebras of operators	173
	4. The concept of a quantum process	181
§ <b>4</b> .	Representation of algebraic structure by commuting operators	183
	1. Representations of a group	183
	2. Generalization of Stone's theorem	184
	3. Representations of a semigroup	193
	4. Representations of a linear space	193
	5. Generalized quantum processes	195
	6. Representations of an algebra	196
	7. Representations of more general structures	197
§5.	Supplementary results on expansions in generalized eigenvectors	198
	1. A converse theorem	199
	2. The case of an imbedding that is not quasinuclear	200
	3. Expansion in eigenfunctions of Carleman operators	203
	4. Expansion in generalized eigenfunctions of operators acting in	
	$L_2$ -spaces	211
	5 Existence of a rigging	213

6. Connection with the theory of commutative normed algebras	
and the nuclear spectral theorem	219
§6. Selfadjointness of operators and uniqueness of the solution of the	
Cauchy problem for evolution equations	225
1 General theorems on the connection between selfadjointness	
and uniqueness	225
2. Some generalizations	231
3. Essential selfadjointness and quasianalytic vectors	237
4. Verification of commutativity for selfadjoint operators	241
5. Parabolic criteria for selfadjointness	246
6. Selfadjointness of perturbations of an operator by a potential	248
Chapter 3. Some Classes of Operators Acting in a Space of Functions	
of Infinitely Many Variables	255
§1. Operators acting with respect to different variables and functions	
of them	258
1. Operators acting with respect to different variables	258
2. The Fourier-Wiener transformation	264
3. Functions of operators acting with respect to different	
variables	<b>268</b>
4. The case of functions which can be well approximated by	
cylindrical functions	<b>27</b> 2
§2. Operators admitting separation of infinitely many variables	276
1. The convolution of finitely many resolutions of the identity	
on the line	276
2. The convolution of infinitely many resolutions of the identity	
on the line and its existence	285
3. Separation of infinitely many variables	294
§3. Differential operators with infinitely many variables	302
<ol> <li>Some facts about general operators with separating variables</li> <li>Differential operators with infinitely many separating</li> </ol>	302
variables	306
3. Differential operators with infinitely many separating	300
variables. The case of annihilation	309
4. First digression. Selfadjointness of differential operators in	309
the case of finitely many variables	313
5. Continuation of the digression: Getting rid of the additional	313
smoothness restrictions on the leading coefficients	322
6. Selfadjointness of differential operators with infintely many	322
variables. The parabolic approach	325
7. Selfadjointness of differential operators with infinitely many	323
variables. Combination of the parabolic approach and the	
hyperbolic approach	335
8. Second digression Operators that are not semihounded below	333 337

CONTENTS	vii
Comments on the literature	343
Bibliography	357
Subject index	381