

Contents

Preface ix

Explanatory Notes xiii

Notation xv

Introduction 3

1 Spaces of Functions and Fourier Transforms 6

§1 Spaces of Functions 6

§2 Linear Operators, Linear Functionals, Dual Spaces 20

§3 The Fourier Transform of Rapidly Decreasing Functions 34

§4 The Fourier Transform of L_1 - and L_2 -Functions 38

§5 The Fourier Transform of Tempered Distributions 40

§6 Oscillatory Integrals 45

2 Pseudo-Differential Operators of Class $S_{\rho,\delta}^m$ 54

§1 Definitions and Algebras 54

§2 Operators with Double Symbols 64

§3 Asymptotic Expansion Formulas 75

§4 L_2 -Boundedness Theorem 79

§5 Construction of a Parametrix 81

§6 Invariance under Coordinate Transformations 88

§7 Pseudo-Differential Operators on a C^∞ -Manifold 95

§8 Pseudo-Differential Operators on a Vector Bundle 106

3 Pseudo-Differential Operators on Sobolev Spaces 116

§1 Pseudo-Differential Operators on \mathcal{S}' and \mathcal{D}' 116

§2 Pseudo-Differential Operators on Sobolev Spaces 118

§3 Hypoellipticity and Local Solvability 126

§4 The Friedrichs Part and a Sharp Form of Gårding's Inequality 128

§5 Fredholm Operators 135

§6 Pseudo-Differential Operators on $H_s(M)$ and $H_s(E)$ 149

§7 The Relation between the Symbol Topology and the Operator Topology 156

4	Second-Order Hypoelliptic Operators	160
§1	Hörmander's Theorem	160
§2	A Pseudo-Differential Operator Formulation	162
§3	An ε -Estimate	165
5	The Index of Fredholm Operators	174
§1	Stability of the Index	174
§2	Noether's Formula	178
6	Boundary-Value Problems for Elliptic Operators	191
§1	A Formulation of Boundary-Value Problems	191
§2	The Sobolev Spaces $H_k(\Omega)$ and the Trace Spaces $H_{k-1/2}(\Gamma)$	196
§3	The Poisson Operator for the Dirichlet Problem	201
§4	The Symbol of the Poisson Operator	210
§5	General Boundary-Value Problem	216
7	Pseudo-Differential Operators of Class $S_{\lambda, \rho, \delta}^m$ and Initial-Value Problems	221
§1	Class $S_{\lambda, \rho, \delta}^m$ Defined by a Basic Weight Function	221
§2	Pseudo-Differential Operators with Multiple Symbols	225
§3	Initial-Value Problems for Hyperbolic Equations	234
§4	Initial-Value Problems for Parabolic Equations	241
8	Complex Powers of Hypoelliptic Operators	248
§1	Definition of the Complex Powers and the Main Theorem	248
§2	Construction of the Complex Powers	252
9	The Fixed-Point Theorem of Atiyah-Bott-Lefschetz	261
§1	Elliptic Complexes and a Fixed-Point Theorem	261
§2	Proof of the Fixed-Point Theorem	267

10	Fourier Integral Operators	276
§1	Fourier Integral Operators on \mathbf{R}^n	276
§2	Product Formulas for $\mathcal{S}_{\rho,\phi}^{m_1} \cdot \mathcal{S}_{\rho}^{m_2}$ and $\mathcal{S}_{\rho}^{m_1} \cdot \mathcal{S}_{\rho,\phi}^{m_2}$	289
§3	Wave Front Sets	300
§4	Fundamental Solutions for Hyperbolic Systems I; Constant Multiplicities	313
§5	Multi-Products of Phase Functions in $\mathcal{P}_{\rho}(\tau)$	331
§6	Multi-Products of Fourier Integral Operators	345
§7	Fundamental Solutions for Hyperbolic Systems II; Variable Multiplicities	360
Appendix		371
I	Existence of Inverses for Pseudo-Differential Operators	371
II	Factorization Theorems	377
III	Systemization Theorems	382
IV	Perfect Diagonalization	391
V	Local Uniqueness of Initial-Value Problems	394
VI	H_{∞} Well-Posedness	405
VII	Bibliographical Notes	410
	Bibliography	421
	Part 1	421
	Part 2	428
	Index	453