

TABLE OF CONTENTS

Preface	ix
Chapter Dependency Diagram	xv
Acknowledgements	xvii
PART I: SOME MAIN RESULTS ON COMMUTATOR IDENTITIES	
Chapter 1. Introduction and Survey	3
1A General Objectives of the Monograph	3
1B Contact with Prior Literature	6
1C The Main Results in Commutation Theory	7
1D The Main Results in Exponentiation Theory	14
1E Results on (Semi) Group-invariant C^∞ -domains	19
1F Typical Applications of Commutation Theory	23
1G Typical Applications of Exponentiation Theory	29
Chapter 2. The Finite-Dimensional Commutation Condition	37
2A Implications of Finite-Dimensionality in Commutation Theory	37
2B Examples involving Differential Operators	40
2C Examples from Universal and Operator Enveloping Algebras	42
2D Relaxing the Finite-Dimensionality Condition	50
PART II: COMMUTATION RELATIONS AND REGULARITY PROPERTIES FOR OPERATORS IN THE ENVELOPING ALGEBRA OF REPRESENTATIONS OF LIE GROUPS	
Introduction	57
Chapter 3. Domain Regularity and Semigroup Commutation Relations	60
3A Lie Algebras of Continuous Operators	62
3B Semigroups and Ad-Orbits	64
3C Variations upon the Regularity Condition	67
3D Infinite-Dimensional $\mathcal{O}_A(B)$	74
Chapter 4. Invariant-Domain Commutation Theory applied to the Mass-Splitting Principle	77
4A Global Invariance/Regularity for Heat-Type Semigroups	79

4B	Formulation of the Generalized Mass-Splitting Theorem	82
4C	The Mass-Operator as a Commuting Difference of Sub-Laplacians	83
4D	Remarks on General Minkowskian Observables	91
4E	Fourier Transform Calculus and Centrality of Isolated Projections	95
PART III: CONDITIONS FOR A SYSTEM OF UNBOUNDED OPERATORS TO SATISFY A GIVEN COMMUTATION RELATION		
	Introduction	101
	Chapter 5. Graph-Density applied to Resolvent Commutation, and Operational Calculus	108
5A	Augmented Spectra and Resolvent Commutation Relations	111
5B	Commutation Relations on D_1	117
5C	Analytic Continuation of Commutation Relations	122
5D	Commutation Relations for the Holomorphic Operational Calculus	124
	Chapter 6. Graph-Density Applied to Semigroup Commutation Relations	131
6A	Semigroup Commutation Relations with a Closable Basis	132
6B	Variants of Sections 5B and 6A for General M	139
6C	Automatic Availability of a Closable Basis	144
6D	Remarks on Operational Calculi	146
	Chapter 7. Construction of Globally Semigroup-invariant C^∞ -domains	150
7A	Fréchet C^∞ -domains in Banach Spaces	151
7B	The Extrinsic Two-Operator Case	155
7C	The Lie Algebra Case	160
7D	C^∞ -action of Resolvents, Projections, and Operational Calculus	164
PART IV: CONDITIONS FOR A LIE ALGEBRA OF UNBOUNDED OPERATORS TO GENERATE A STRONGLY CONTINUOUS REPRESENTATION OF THE LIE GROUP		
	Introduction	173
	Chapter 8. Integration of Smooth Operator Lie Algebras	177
8A	Smooth Lie Algebras and Differentiable Representations	178
8B	Applications in C^∞ -vector spaces	186
	Chapter 9. Exponentiation and Bounded Perturbation of Operator Lie Algebras	194
9A	Discussion of Exponentiation Theorems and Applications	194

9B	Proofs of the Theorems	199
9C	Phillips Perturbations of Operator Lie Algebras and Analytic Continuation of Group Representations	206
9D	Semidirect Product Perturbations	217
	Appendix to Part IV	227
PART V: LIE ALGEBRAS OF VECTOR FIELDS ON MANIFOLDS		
	Introduction	235
	Chapter 10. Applications of Commutation Theory to Vector-Field Lie Algebras and Sub- Laplacians on Manifolds	240
10A	Exponentials versus Geometric Integrals of Vector-Field Lie Algebras	243
10B	Exponentiation on L^p spaces	251
10C	Sub-Laplacians on Manifolds	265
10D	Solution Kernels on Manifolds	268
PART VI: DERIVATIONS ON MODULES OF UNBOUNDED OPERATORS WITH APPLICATIONS TO PARTIAL DIFFERENTIAL OPERATORS ON RIEMANN SURFACES		
	Introduction	277
	Chapter 11. Rigorous Analysis of Some Commutator Identities for Physical Observables	279
11A	Variations upon the Graph-Density and Kato Conditions	282
11B	Various forms of Strong Commutativity	287
11C	Nilpotent Commutation Relations of Generalized Heisenberg-Weyl Type	302
	Appendix to Part VI	320
PART VII: LIE ALGEBRAS OF UNBOUNDED OPERATORS: PERTURBATION THEORY, AND ANALYTIC CONTINUATION OF $\mathfrak{sl}(2, \mathbb{R})$ -MODULES		
	Introduction	331
	Chapter 12. Exponentiation and Analytic Continuation of Heisenberg-Matrix Representations for $\mathfrak{sl}(2, \mathbb{R})$	335
12A	Connections to the Theory of TCI Representations of Semisimple Groups on Banach Spaces	339
12B	The Graph-Density Condition and Base-Point Exponentials	355
12C	C^∞ -integrals and Smeared Exponentials on \mathfrak{l}^p	365
12D	The Operators $A_0, A_1,$ and A_2	376
12E	Compact and Phillips Perturbations	389

12F Perturbations and Analytic Continuation of Smearred Representations	399
12G Irreducibility, Equivalences, Unitarity, and Single-Valuedness	407
12H Perturbation and Reduction Properties of Other Analytic Series	423
12I A Counter-Theorem on Group-Invariant Domains	429
Appendix to Part VII	432
GENERAL APPENDICES	
Appendix A. The Product Rule for Differentiable Operator Valued Mappings	439
Appendix B. A Review of Semigroup Folklore, and Integration in Locally Convex Spaces	443
Appendix C. The Square of an Infinitesimal Group Generator	451
Appendix D. An Algebraic Characterization of $\mathcal{O}_{A_2}^{(B)}$	457
Appendix E. Compact Perturbations of Semigroups	461
Appendix F. Numerical Ranges, and Semigroups on L^p spaces	465
Appendix G. Bounded Elements in Operator Lie Algebras	470
References	476
References to Quotations	486
Index	487
List of Symbols	491