

Contents

Preface	vii
List of Notations	xv

PART I. CLIFFORD ALGEBRAS AND DIRAC OPERATORS

1. Clifford Algebras and Clifford Modules	3
2. Clifford Bundles and Compatible Connections	10
3. Dirac Operators	19
4. Dirac Laplacian and Connection Laplacian	26
5. Euclidean Examples	29
6. The Classical Dirac (Atiyah-Singer) Operators on Spin Manifolds	36
7. Dirac Operators and Chirality	40
8. Unique Continuation Property for Dirac Operators	43
9. Invertible Doubles	50
10. Glueing Constructions. Relative Index Theorem	59

PART II. ANALYTICAL AND TOPOLOGICAL TOOLS

11. Sobolev Spaces on Manifolds with Boundary	67
12. Calderón Projector for Dirac Operators	75
13. Existence of Traces of Null Space Elements	95
14. Spectral Projections of Dirac Operators	105
15. Pseudo-Differential Grassmannians	111
16. The Homotopy Groups of the Space of Self-Adjoint Fredholm Operators A. Elementary Decompositions and Deformations	127
B. The Homotopy Groups of \widehat{G}_*	133
17. The Spectral Flow of Families of Self-Adjoint Operators A. Continuity of Eigenvalues	138
B. The Spectral Flow on Loops in $\widehat{\mathcal{F}}_*$	140
C. Spectral Flow and Index	145
D. Non-Vanishing Spectral Flow	157

PART III. APPLICATIONS

18. Elliptic Boundary Problems and Pseudo-Differential Projections	163
19. Regularity of Solutions of Elliptic Boundary Problems	180

20. Fredholm Property of the Operator A_R	188
21. Exchanges on the Boundary: Agranovič-Dynin Type Formulas and the Cobordism Theorem for Dirac Operators .	205
22. The Index Theorem for Atiyah-Patodi-Singer Problems	
A. Preliminary Remarks	211
B. Heat Kernels on the Cylinder	214
C. Duhamel's principle. Heat Kernels on Manifolds with Boundary	231
D. Proof of the Index Formula	239
E. L^2 -Reformulation	242
F. The Odd-Dimensional Case. A Three-Dimensional Example	248
23. Some Remarks on the Index of Generalized Atiyah-Patodi-Singer Problems	253
24. Bojarski's Theorem. General Linear Conjugation Problems	262
25. Cutting and Pasting of Elliptic Operators	276
26. Dirac Operators on the Two-Sphere	282
Bibliography	289
Index	303