

# Contents

<b>Series Preface</b>	<b>v</b>
<b>Preface</b>	<b>vii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 What Is Mathematical Control Theory? . . . . .	1
1.2 Proportional-Derivative Control . . . . .	2
1.3 Digital Control . . . . .	6
1.4 Feedback Versus Precomputed Control . . . . .	9
1.5 State-Space and Spectrum Assignment . . . . .	11
1.6 Outputs and Dynamic Feedback . . . . .	17
1.7 Dealing with Nonlinearity . . . . .	21
1.8 A Brief Historical Background . . . . .	22
1.9 Some Topics Not Covered . . . . .	24
<b>2 Systems</b>	<b>25</b>
2.1 Basic Definitions . . . . .	25
2.2 I/O Behaviors . . . . .	30
2.3 Discrete-Time . . . . .	33
2.4 Linear Discrete-Time Systems . . . . .	36
2.5 Smooth Discrete-Time Systems . . . . .	39
2.6 Continuous-Time . . . . .	42
2.7 Linear Continuous-Time Systems . . . . .	47
2.8 Linearizations Compute Differentials . . . . .	53
2.9 More on Differentiability* . . . . .	62
2.10 Sampling . . . . .	70
2.11 Volterra Expansions* . . . . .	71
2.12 Notes and Comments . . . . .	74

<b>3</b>	<b>Reachability and Controllability</b>	<b>79</b>
3.1	Basic Reachability Notions . . . . .	79
3.2	Time-Invariant Systems . . . . .	82
3.3	Controllable Pairs of Matrices . . . . .	90
3.4	Controllability Under Sampling . . . . .	97
3.5	More on Linear Controllability . . . . .	102
3.6	First-Order Local Controllability . . . . .	115
3.7	Piecewise Constant Controls . . . . .	125
3.8	Notes and Comments . . . . .	126
<b>4</b>	<b>Feedback and Stabilization</b>	<b>131</b>
4.1	Constant Linear Feedback . . . . .	131
4.2	Feedback Equivalence* . . . . .	137
4.3	Disturbance Rejection and Invariance* . . . . .	146
4.4	Stability and Other Asymptotic Notions . . . . .	149
4.5	Unstable and Stable Modes* . . . . .	154
4.6	Lyapunov's Direct Method . . . . .	157
4.7	Linearization Principle for Stability . . . . .	170
4.8	More on Smooth Stabilizability* . . . . .	173
4.9	Notes and Comments . . . . .	185
<b>5</b>	<b>Outputs</b>	<b>189</b>
5.1	Basic Observability Notions . . . . .	189
5.2	Time-Invariant Systems . . . . .	197
5.3	Continuous-Time Linear Systems . . . . .	204
5.4	Linearization Principle for Observability . . . . .	208
5.5	Realization Theory for Linear Systems . . . . .	211
5.6	Recursion and Partial Realization . . . . .	219
5.7	Rationality and Realizability . . . . .	225
5.8	Abstract Realization Theory* . . . . .	232
5.9	Notes and Comments . . . . .	239
<b>6</b>	<b>Observers and Dynamic Feedback</b>	<b>243</b>
6.1	Observers and Detectability . . . . .	243
6.2	Dynamic Feedback . . . . .	249
6.3	External Stability for Linear Systems . . . . .	254
6.4	Frequency-Domain Considerations . . . . .	258
6.5	Parameterization of Stabilizers . . . . .	264
6.6	Notes and Comments . . . . .	271

<b>7</b>	<b>Optimal Control</b>	<b>273</b>
7.1	An Optimal Control Problem . . . . .	275
7.2	Dynamic Programming . . . . .	276
7.3	The Continuous-Time Case . . . . .	284
7.4	Linear Systems with Quadratic Cost . . . . .	289
7.5	Infinite-Time Problems . . . . .	295
7.6	Tracking . . . . .	306
7.7	(Deterministic) Kalman Filtering . . . . .	310
7.8	Notes and Comments . . . . .	316
	<b>Appendixes</b>	<b>319</b>
<b>A</b>	<b>Linear Algebra</b>	<b>319</b>
A.1	Operator Norms . . . . .	319
A.2	Singular Values . . . . .	320
A.3	Jordan Forms and Matrix Functions . . . . .	324
A.4	Continuity of Eigenvalues . . . . .	328
<b>B</b>	<b>Differentials</b>	<b>333</b>
B.1	Finite Dimensional Mappings . . . . .	333
B.2	Maps Between Normed Spaces . . . . .	335
<b>C</b>	<b>Ordinary Differential Equations</b>	<b>339</b>
C.1	Review of Lebesgue Measure Theory . . . . .	339
C.2	Initial-Value Problems . . . . .	344
C.3	Existence and Uniqueness Theorem . . . . .	346
C.4	Continuous Dependence . . . . .	357
C.5	Linear Differential Equations . . . . .	359
C.6	Stability of Linear Equations . . . . .	363
	<b>Bibliography</b>	<b>365</b>
	<b>Index</b>	<b>389</b>