Contents

Preface		•
1	Unilateral Problems for Scalar Functions	
1.1	Unilateral Boundary Value Problems for	•
	Second Order Equations	2
1.1.1	Primal and Dual Variational Problems	4
1.1.11	Dual Variational Formulation	•
1.1.12	Relation Between the Primal and Dual	
	Variational Formulations	10
1.1.2	Mixed Variational Formulations	13
1.1.3	Solution of Primal Problems by the	
	Finite Element Method and Error Bounds	18
1.1.31	Approximation of Problem P_1 by	
	the Finite Element Method	18
1.1.32	The General Theory of Approximations for	
	Elliptic Inequalities	21
1.1.33	A Priori Bound for Problem P ₁	24
1.1.4	Solution of Dual Problems by the	
	Finite Element Method and Error Bounds	27
1.1.41	Problems with Absolute Terms	27
1.1.411	A Priori Error Bounds	29
1.1.412	A Posteriori Error Bounds and the	
	Two-Sided Energy Bound	34
1.1.42	••	36
1.1.421	A Priori Error Bound	41
	A Destruit Proce Describer 141	* 1

viii	Contents
------	----------

	Two-Sided Energy Estimate	48
1.1.5	Solution of Mixed Problems by the Finite	
	Element Method and Error Bounds	49
1.1.51	Mixed Variational Formulations of Elliptic	
	Inequalities	51
1.1.52	Approximation of the Mixed Variational	
	Formulation and Error Bounds	54
1.1.53	Numerical Realization of Mixed	
	Variational Formulations	59
1.1.6	Semicoercive Problems	62
1.1.61	Solution of the Primal Problem by the Finite	
	Element Method and Error Bounds	66
1.1.62	Solution of the Dual Problem by the Finite	
	Element Method and Error Bounds	69
1.1.63	Convergence of the Dual Finite Element Method	75
1.1.7	Problems with Nonhomogeneous Boundary Obstacle	82
1.1.71	Approximation of the Primal Problem	85
1.1.72		-
	Finite Element Method	87
1.1.73	A Posteriori Error Bounds and Two-Sided	-
	Energy Estimate	88
1.2	Problems with Inner Obstacles for	
	Second-Order Operators	89
1.2.1	Primal and Dual Variational Formulations	89
1.2.2	Mixed Variational Formulation	93
1.2.3	Solution of the Primal Problem by the	
	Finite Element Method	94
1.2.4	Solution of the Dual Problem by the	
	Finite Element Method	98
1.2.41	Approximation of the Dual Formulation of the	
	Problem with an Inner Obstacle	98
1.2.42	Construction of the Sets Q_{fh}^- and	
	Their Approximate Properties	99
1.2.43	A Priori Error Bound of the Approximation	30
	of the Dual Formulation	99
1.2.5	Solution of the Mixed Formulation by the	
	Finite Element Method	104

Contents ix

2	One-Sided Contact of Elastic Bodies	109
2.1	Formulations of Contact Problems	110
2.1.1	Problems with Bounded Zone of Contact	112
2.1.2	Problems with Increasing Zone of Contact	114
2.1.3	Variational Formulations	116
2.2	Existence and Uniqueness of Solution	121
2.2.1	Problem with Bounded Zone of Contact	121
2.2.2	Problem with Increasing Zone of Contact	130
2.3	Solution of Primal Problems by the Finite	
	Element Method	134
2.3.1	Approximation of the Problem with a Bounded	
	Zone of Contact	134
2.3.2	Approximation Problems with Increasing	
	Zone of Contact	136
2.3.3	A Priori Error Estimates and the Convergence	138
2.3.31	Bounded Zone of Contact	138
	Polygonal Domains	139
	Curved Contact Zone	148
2.3.32	Increasing Zone of Contact	154
2.4	Dual Variational Formulation of the Problem with	
	Bounded Zone of Contact	164
2.4.1	Approximation of the Dual Problem	170
2.4.11	Equilibrium Model of Finite Elements	173
2.4.12	Applications of the Equilibrium Model	175
2.4.13	Algorithm for Approximations of the	
	Dual Problem	176
2.5	Contact Problems with Friction	182
2.5.1	The Problem with Prescribed Normal Force	187
2.5.2	Some Auxiliary Spaces	192
2.5.3	Existence of Solution of the Problem with Friction	194
2.5.4	Algorithms for the Contact Problem with	
	Friction for Elastic Bodies	196
2.5.41	Direct Iterations	196
2.5.42	Alternating Iterations	207
2.5.421	Unilateral Contact with a Given Shear Force	208
2.5.422	Realizability of the Algorithm of Alternating Iterations	212

х		Contents

3	Problems of the Theory of Plasticity	221
3.1	Prandtl-Reuss Model of Plastic Flow	226
3.1.1	Existence and Uniqueness of Solution	229
3.1.2	Solution by Finite Elements	233
3.1.21	A Priori Error Estimates	235
3.2	Plastic Flow with Isotropic or Kinematic Hardening	238
3.2.1	Existence and Uniqueness of Solution of the	
	Plastic Flow Problem with Hardening	241
3.2.2	Solution of Isotropic Hardening by Finite Elements	247
3.2.21	A Priori Error Estimates	250
3.2.22	A Priori Error Estimates for the Plane Problem	258
3.2.23	Convergence in the Case of Nonregular Solution	262
Referer	nces	267
Index		273