

Contents

<i>Preface</i>	<i>page ix</i>
Part One: Poisson geometry and morita equivalence	1
1 Introduction	3
2 Poisson geometry and some generalizations	5
2.1 Poisson manifolds	5
2.2 Dirac structures	7
2.3 Twisted structures	11
2.4 Symplectic leaves and local structure of Poisson manifolds	13
2.5 Presymplectic leaves and Dirac manifolds	15
2.6 Poisson maps	18
2.7 Dirac maps	20
3 Algebraic Morita equivalence	25
3.1 Ring-theoretic Morita equivalence of algebras	25
3.2 Strong Morita equivalence of C^* -algebras	29
3.3 Morita equivalence of deformed algebras	33
4 Geometric Morita equivalence	37
4.1 Representations and tensor product	37
4.2 Symplectic groupoids	40
4.3 Morita equivalence for groups and groupoids	47
4.4 Modules over Poisson manifolds and groupoid actions	49
4.5 Morita equivalence and symplectic groupoids	52
4.6 Picard groups	58
4.7 Fibrating Poisson manifolds and Morita invariants	61
4.8 Gauge equivalence of Poisson structures	64
5 Geometric representation equivalence	67
5.1 Symplectic torsors	67
5.2 Symplectic categories	69

5.3	Symplectic categories of representations	70
	<i>Bibliography</i>	72
Part Two: Formality and star products		79
1	Introduction	81
1.1	Physical motivation	81
1.2	Historical review of deformation quantization	83
1.3	Plan of the work	85
2	The star product	87
3	Rephrasing the main problem: the formality	93
3.1	DGLA's, L_∞ -algebras and deformation functors	94
3.2	Multivector fields and multidifferential operators	102
3.2.1	The DGLA \mathcal{V}	103
3.2.2	The DGLA \mathcal{D}	106
3.3	The first term: U_1	111
4	Digression: what happens in the dual	113
5	The Kontsevich formula	120
5.1	Admissible graphs, weights and B_Γ 's	121
5.2	The proof: Stokes' theorem & Vanishing theorems	125
6	From local to global deformation quantization	134
	<i>Bibliography</i>	141
Part Three: Lie groupoids, sheaves and cohomology		145
1	Introduction	147
2	Lie groupoids	149
2.1	Lie groupoids and weak equivalences	151
2.2	The monodromy and holonomy groupoids of a foliation	154
2.3	Étale groupoids and foliation groupoids	156
2.4	Some general constructions	159
2.5	Principal bundles as morphisms	164
2.6	The principal bundles category	168
3	Sheaves on Lie groupoids	175
3.1	Sheaves on groupoids	176
3.2	Functoriality and Morita equivalence	182
3.3	The fundamental group and locally constant sheaves	187
3.4	G -sheaves of R -modules	201
3.5	Derived categories	205
4	Sheaf cohomology	210
4.1	Sheaf cohomology of foliation groupoids	211
4.2	The bar resolution for étale groupoids	214
4.3	Proper maps and orbifolds	221

4.4	A comparison theorem for foliations	227
4.5	The embedding category of an étale groupoid	232
4.6	Degree one cohomology and the fundamental group	238
5	Compactly supported cohomology	242
5.1	Sheaves over non-Hausdorff manifolds	243
5.2	Compactly supported cohomology of étale groupoids	249
5.3	The operation $\phi_!$	254
5.4	Leray spectral sequence, and change-of-base	258
5.5	Homology of the embedding category	264
	<i>Bibliography</i>	269
Part Four: Geometric methods in representation theory		273
1	Reductive Lie Groups: Definitions and Basic Properties	275
1.1	Basic Definitions and Examples	275
1.2	The Cartan Decomposition	276
1.3	Complexifications of Linear Groups	279
2	Compact Lie Groups	282
2.1	Maximal Tori, the Unit Lattice, and the Weight Lattice	282
2.2	Weights, Roots, and the Weyl Group	284
2.3	The Theorem of the Highest Weight	286
2.4	Borel Subalgebras and the Flag Variety	289
2.5	The Borel-Weil-Bott Theorem	291
3	Representations of Reductive Lie Groups	294
3.1	Continuity, Admissibility, $K_{\mathbb{R}}$ -finite and C^∞ Vectors	294
3.2	Harish-Chandra Modules	298
4	Geometric Constructions of Representations	305
4.1	The Universal Cartan Algebra and Infinitesimal Characters	306
4.2	Twisted \mathcal{D} -modules	307
4.3	Construction of Harish-Chandra Modules	311
4.4	Construction of $G_{\mathbb{R}}$ -representations	314
4.5	Matsuki Correspondence	317
	<i>Bibliography</i>	321
Part Five: Deformation theory: a powerful tool in physics modelling		325
1	Introduction	327
1.1	It ain't necessarily so	327
1.2	Epistemological importance of deformation theory	328

2 Composite elementary particles in AdS microworld	332
2.1 A qualitative overview	333
2.2 A brief overview of singleton symmetry & field theory	335
3 Nonlinear covariant field equations	338
4 Quantisation is a deformation	340
4.1 The Gerstenhaber theory of deformations of algebras	340
4.2 The invention of deformation quantisation	342
4.3 Deformation quantisation and its developments	345
<i>Bibliography</i>	348
<i>Index</i>	355