

Contents

Introduction: A brief travel guide to the journey ahead	xi
To the student: A great adventurexii
To the instructor: The mathematics to come.	xiv
1. Puzzles and patterns: A precursor to proofs.	1
Senators and snakes	2
Checkerboards undercover.	3
Flipping cards and coins	6
Juggling balls	7
Counting on a good hand and a good grade	8
Stepping back	10
2. Bringing theorems to justice: Exposing the truth through rigorous proof	11
Making a statement	11
The truth, the whole truth, and other stuff besides the truth	13
Arguing the case through proof	16
The domino effect of bringing everyone down through guilt by induction	19
Stepping back	26
3. Delving into the dependable digits: Counting on counting numbers	27
Divide and conquer	27
Solving equations by simple division	28
Now ready for prime time	31
Stepping back	32

4. Going around in circles: The art of modular arithmetic.	33
The mod side of mathematics	33
Old-school algebra gets an extreme makeover	35
The power of congruences	36
Public secret codes	37
Stepping back	39
5. The irrational side of numbers: A world of nonrepeating digits.	41
Divisible and indivisible.	41
Forgoing fractions.	42
Irrationalit- <i>e</i>	43
Stepping back	43
6. Discovering how to function in mathematics:	
Moving beyond ordinary relations	45
Get ready, get set	45
Fun with functions	47
Moving onto an intimate one-to-one relationship with functions.	48
An intuitively-believable-yet-challenging-to-verify correspondence	50
Stepping back	51
7. Infinity: Understanding the unending.	53
Comparing cardinality	53
Countable collections	54
The return of power sets	55
A pair of paradoxes	56
Stepping back	56
8. Recursively defined functions: The next generation.	57
Moving ahead by looking back	57
Forgoing convergence—A formal look at power series	58
Generating closed formulas through generating functions	60
Stepping back	63

9. Discrete thoughts of counting: Quantifying complicated quantities	65
Pigeons without a home.	65
PERMUTATION = UP TO A REMINT	66
A combination that unlocks counting	67
Stepping back	69
10. Quantifying uncertainty with probability: A likely story?	71
What are the chances?	71
A coin toss?	73
What's the deal?	73
Stepping back	74
11. The subtle art of connecting the dots: Edging up to graphs	75
Bridging the graph	75
Making the circuit	77
Branching out to trees	79
Completely connected graphs	80
Stepping back	81
12. Just plane graphs: Drawing without being cross	83
Drawing without crossing	83
Seemingly similar graphs	85
Sensing symmetry.	88
Stepping back	89
13. Visible and invisible universes: Geometric vignettes	91
Sizing up symmetry	91
Keeping an artful eye on art galleries	95
Up, up, and away—The fourth dimension	97
Stepping back	100
14. A synergy between geometry and numbers:	
Circles and Pythagorean triples	101
Rightful triangles	101
Determining which triangles are allright	102
A rational look at the circle.	103
Stepping back	104

15. The mathematical mysteries within a sheet of paper:	
Unfolding pattern and structure	105
Getting into the fold	105
The story unfolds	106
Adding arithmetic to generate a wild parity	107
Stepping back	108
16. Take it to the limit: An initial approach to analysis	109
Getting really close	109
Grasping the cloudlike Cantor set	111
Turning to ternary	112
Stepping back	114
17. Uninterrupted thoughts of continuity: A jump-free journey	115
Functions without lifting a finger	115
Valuing intermediate values	116
A devilish staircase	117
Stepping back	119
18. An abstract world of algebra: Reconciling with your x	121
Grasping at groups	122
A group mentality	123
Building a group table	124
Stepping back	125
19. Cycles and curves: Algebraic structure in numbers and geometry	127
The subculture of subgroups	127
Cyclic groups	128
A circular group	128
Stepping back	131
20. Further frontiers	133

Appendix 1. Hints, remarks, and leading questions	139
Appendix 2. A proof primer: The proof is in the pudding	151
1. Preproof prep	151
2. Penning the proof	152
3. Post proof	153
Appendix 3. Commentary for instructors	155
How I used this material in my classroom	157
Appendix 4. A math personality questionnaire	161
Acknowledgments	163
Index	165