

# Contents

## CHAPTER 1

### General Facts About Probability Distributions

§1. Probability Spaces	1
1. Measurable Spaces	1
2. Distributions and Measures	3
3. Probability Spaces	8
§2. Conditional Distributions	12
1. Conditional Expectation	12
2. Conditional Probability Distributions	14
§3. Zero-One Laws. Regularity	19
1. Zero-One Law	19
2. Decomposition Into Regular Components	21
§4. Consistent Conditional Distributions	22
1. Consistent Conditional Distributions for a Given Probability Measure	22
2. Probability Measures with Given Conditional Distributions	23
3. Construction of Consistent Conditional Distributions	27
§5. Gaussian Probability Distributions	36
1. Basic Definitions and Examples	36
2. Some Useful Propositions	41
3. Gaussian Linear Functionals on Countably-Normed Hilbert Spaces	43
4. Polynomials of Gaussian Variables and Their Conditional Expectations	47
5. Hermite Polynomials and Multiple Stochastic Integrals	51

## CHAPTER 2

### Markov Random Fields

§1. Basic Definitions and Useful Propositions	55
1. Splitting $\sigma$ -algebras	55
2. Markov Random Processes	58

3. Random Fields; Markov Property	62
4. Transformations of Distributions which Preserve the Markov Property. Additive Functionals	69
<b>§2. Stopping <math>\sigma</math>-algebras. Random Sets and the Strong Markov Property</b>	<b>74</b>
1. Stopping $\sigma$ -algebras	74
2. Random Sets	79
3. Compatible Random Sets	82
4. Strong Markov Property	85
<b>§3. Gaussian Fields. Markov Behavior in the Wide Sense</b>	<b>92</b>
1. Gaussian Random Fields	92
2. Splitting Spaces	93
3. Markov Property	95
4. Orthogonal Random Fields	98
5. Dual Fields. A Markov Criterion	99
6. Regularity Condition. Decomposition of a Markov Field into Regular and Singular Components	100

## CHAPTER 3

### The Markov Property for Generalized Random Functions

<b>§1. Biorthogonal Generalized Functions and the Duality Property</b>	<b>103</b>
1. The Meaning of Biorthogonality for Generalized Functions in Hilbert Space	103
2. Duality of Biorthogonal Functions	106
3. The Markov Property for Generalized Functions	112
<b>§2. Stationary Generalized Functions</b>	<b>113</b>
1. Spectral Representation of Coupled Stationary Generalized Functions	113
2. Biorthogonal Stationary Functions	117
3. The Duality Condition and a Markov Criterion	119
<b>§3. Biorthogonal Generalized Functions Given by a Differential Form</b>	<b>122</b>
1. Basic Definitions	122
2. Conditions for Markov Behavior	125
<b>§4. Markov Random Functions Generated by Elliptic Differential Forms</b>	<b>129</b>
1. Levy Brownian Motion	129
2. Structure of Spaces for Given Elliptic Forms	133
3. Boundary Conditions	139
4. Regularity and the Dirichlet Problem	145
<b>§5. Stochastic Differential Equations</b>	<b>150</b>
1. Markov Transformations of "White Noise"	150
2. The Interpolation and Extrapolation Problems	155
3. The Brownian Sheet	157

## CHAPTER 4

<b>Vector-Valued Stationary Functions</b>	<b>163</b>
§1. Conditions for Existence of the Dual Field	163
1. Spectral Properties	163
2. Duality	166
§2. The Markov Property for Stationary Functions	173
1. The Markov Property When a Dual Field Exists	173
2. Analytic Markov Conditions	175
§3. Markov Extensions of Random Processes	180
1. Minimal Nonanticipating Extension	180
2. Markov Stationary Processes	182
3. Stationary Processes with Symmetric Spectra	186
<b>Notes</b>	<b>191</b>
<b>Bibliography</b>	<b>195</b>
<b>Index</b>	<b>199</b>