

# CONTENTS

PREFACE		v
Chapter 1	INTRODUCTION	1
1.1	Complex systems	1
1.2	Statistical Theory of Levels	2
1.3	Comparison with Statistical Mechanics	3
1.4	Examples	4
1.5	Distribution of the Widths	7
1.6	Preview	8
Chapter 2	HIGHLY EXCITED SYSTEMS	9
2.1	One-Dimensional Systems	9
2.2	Sequences of Numbers	10
2.3	Quantal Spectra	12
Chapter 3	SYMMETRY PROPERTIES OF PHYSICAL SYSTEMS	15
3.1	Symmetry Properties	15
3.2	Time-Reversal	16
3.3	The Hamiltonian Matrix	18
3.4	Relation to Kramers' Degeneracy	19
3.5	Quaternion Structure	20
3.6	The Threefold Way of Invariance	21

Chapter 4	GAUSSIAN AND ORTHOGONAL ENSEMBLES	23
4.1	The Gaussian Ensemble	23
4.2	The Orthogonal Ensemble	24
4.3	The Semicircle Distribution	25
4.4	The Wishart Ensemble	26
4.5	Invariance and Equal Probability	27
Chapter 5	UNITARY ENSEMBLE	29
5.1	Systems without Time-Reversal Symmetry	29
5.2	Level Repulsion in Spectra	31
5.3	Repulsion in Low Energy Spectra	39
Chapter 6	EIGENVALUE-EIGENVECTOR DISTRIBUTIONS OF THE GAUSSIAN ENSEMBLE	43
6.1	The Gaussian Ensemble	43
6.2	The Repulsion of Levels	46
6.3	Wigner's Conjecture	47
6.4	Mean Level Density	49
6.5	Remark	50
6.6	The Spectral Rigidity	50
Chapter 7	DISTRIBUTION OF THE WIDTHS	57
7.1	Widths of Energy Levels	57
7.2	The Porter-Thomas Distribution	58
7.3	Strength Fluctuations and Collective Behavior	60
7.4	The Porter-Thomas Distribution and Experimental Data	62
Chapter 8	SYMPLECTIC GROUP AND QUATERNIONS	67
8.1	The Symplectic Group	67
8.2	Quaternions	68
8.3	Matrices and Quaternions	69
8.4	Quaternion Algebra	70
8.5	Symplectic Ensemble	72
Chapter 9	MORE ON THE GAUSSIAN ENSEMBLE	75
9.1	The Gaussian Ensemble	75
9.2	The Distribution Function	76
9.3	The Jacobian of the Transformation	78
9.4	Specification of the Distribution Function	79
9.5	The Unitary Ensemble Case	80

**CONTENTS**

ix

Chapter 10	SUMMARY	83
10.1	Random Matrices and Energy Levels	83
10.2	Role of the Hamiltonian	84
10.3	The Wishart Distribution	85
10.4	Recent Studies	85
10.5	Finite and Infinite Matrices	86
Appendix A	MULTIVARIATE DISTRIBUTIONS	87
A1.	Distributions	87
A2.	Preliminaries	88
A3.	Distributions of Some Random Matrices	94
A4.	Marginal Distributions of Few Roots	100
A5.	Moments of the Elementary Symmetric Functions	104
A6.	Distributions of the Ratios of the Roots	108
A7.	Distributions of the Likelihood Ratio Test Statistics	109
Appendix B	APPLICATIONS OF MULTIVARIATE DISTRIBUTIONS	115
B1.	Preliminaries	115
B2.	Testing Hypotheses	116
B3.	Discrimination between Multivariate Normal Population	122
B4.	Alternative Procedures	123
Appendix C	RANDOM MATRICES: ERGODIC PROPERTIES	129
C1.	Preliminaries	130
C2.	One-Point Measures	135
C3.	Higher-Order Functions	139
C4.	Behavior of the Strength Distribution	146
C5.	Final Remarks	150
REFERENCES		151
BIBLIOGRAPHY		169
INDEX		179