

---

**TABLE OF CONTENTS**


---

**PART A: INTRODUCTION****Chapter I: Branching phenomena and models**

1. Simple branching processes .....	2
2. $p$ -type processes .....	7
3. Age dependence .....	11
4. General processes .....	13
Bibliographical notes .....	16

**PART B: SIMPLE BRANCHING PROCESSES****Chapter II: The Galton-Watson process: Probabilistic methods**

1. Introduction .....	18
2. The Kesten-Stigum theorem .....	23
3. Finer limit theorems: Finite offspring variance .....	28
4. Finer limit theorems: Infinite offspring variance .....	36
5. The Seneta-Heyde theorem .....	43
6. Immigration .....	50
Bibliographical notes .....	54

**Chapter III: The Galton-Watson process: Analytic methods**

1. Subcritical processes: Yaglom's theorem .....	56
2. Arbitrary initial distributions and invariant measures ...	65
3. Critical processes: The exponential limit theorem .....	74
4. Local limit theorems for critical processes .....	78
5. Supercritical processes: Basic convergence result .....	83
6. Further properties of the limiting distribution .....	89
7. Local limit theorem for supercritical processes .....	97
8. Immigration .....	106
Bibliographical notes .....	113

**Chapter IV: Continuous time Markov branching processes**

1. Introduction .....	116
2. Generating functions in continuous time .....	119
3. The method of discrete skeletons .....	127
4. Split times .....	132
Bibliographical notes .....	136

PART C: MULTIGROUP BRANCHING DIFFUSIONS ON BOUNDED DOMAINS

Chapter V: Foundations

1. Existence and construction .....	138
2. Generating functionals and moments .....	149
3. Examples .....	163
4. Equivalence of moment conditions .....	176
Bibliographical notes .....	182

Chapter VI: Limit theory for subcritical and critical processes

1. Subcritical processes with initial distribution concentrated at one point .....	184
2. Subcritical processes with arbitrary initial distributions and invariant measures .....	193
3. Critical processes with finite second moment parameter ...	200
4. Critical processes with infinite second moment parameter .	207
5. Critical processes without proper conditional limit .....	218
6. Subcritical and critical processes with immigration .....	226
Bibliographical notes .....	240

Chapter VII: Basic limit theory for supercritical processes

1. Extinction probability and transience .....	242
2. Normalizing constants .....	249
3. Extinction and transience continued .....	257
4. Properties of the limiting distribution .....	263
5. Almost sure convergence with general test functions .....	269
6. Supercritical processes with immigration .....	279
Bibliographical notes .....	288

Chapter VIII: More on the limiting behaviour of linear functionals

1. Introduction .....	290
2. Interlude on the Jordan canonical form .....	293
3. Introduction continued .....	296
4. Interlude on second moments .....	299
5. Exploiting the additivity property .....	305
6. Exploiting martingales: A central limit theorem for triangular arrays .....	313
7. Exploiting martingales: The increasing process and the central limit theorem for linear functionals .....	316

8. Exploiting martingale exponential inequalities .....	321
9. The law of the iterated logarithm in the case $\lambda^2 = \rho$ .....	328
10. The case $\lambda^2 > \rho$ .....	331
11. An example from asymptotic estimation theory .....	334
12. Continuous time .....	337
13. Infinite variance .....	343
Bibliographical notes .....	346

#### PART D: RELATED MODELS

##### Chapter IX: Unbounded domains

1. The branching Ornstein-Uhlenbeck processes .....	348
2. Branching Brownian motion .....	354
Bibliographical notes .....	359

##### Chapter X: Generalized age-dependence and random characteristics

1. Introduction .....	362
2. Renewal techniques .....	364
3. Age-dependence, the stable age-distribution, and the reproductive value .....	366
4. Martingales and a theorem of Kesten-Stigum type .....	371
5. Empirical ratio limit theorems in the supercritical case ..	375
6. The subcritical case .....	380
7. The critical case .....	387
8. Multitype generalizations .....	398
Bibliographical notes .....	402

##### Chapter XI: Two-sex models

1. Models and examples .....	404
2. Limit theorems for non-overlapping generations .....	409
3. Limit theorems for overlapping generations: The determi- nistic differential equations .....	415
4. Limit theorems for overlapping generations: The pure birth process .....	420
Bibliographical notes .....	427

#### APPENDIX

1. The conditional Borel-Cantelli lemma .....	430
2. Martingale convergence theorems .....	431

3. Uniform integrability .....	432
4. Series with independent terms .....	433
5. Summation by parts .....	433
6. Maximal inequalities .....	434
7. Results related to the LIL .....	435
8. The martingale CLT .....	438
9. The Croft-Kingman lemma .....	438
10. Results related to the LINN .....	439
11. A result of Anscombe-Renyi type .....	441
12. A weak LLN rate of convergence result .....	444
13. Slowly or regularly varying functions .....	445
14. Tauberian theorems .....	450
REFERENCES .....	454

FIRST APPEARANCE OF SPECIALLY MARKED RELATIONS:

(F.1), (F.2)	page 139
(IF)	147
(IM)	155
(M)	156
(FM)	159
(RM)	159
(R)	160
(C), (C*)	160
(T.1) - (T.5)	164, 165
(S)	207