

Contents

	Preface	xv
Chapter 1	P. Bézier: How a Simple System Was Born	1
Chapter 2	Introductory Material	13
	2.1 Points and Vectors	13
	2.2 Affine Maps	17
	2.3 Constructing Affine Maps	20
	2.4 Function Spaces	22
	2.5 Problems	24
Chapter 3	Linear Interpolation	25
	3.1 Linear Interpolation	25
	3.2 Piecewise Linear Interpolation	29
	3.3 Menelaos' Theorem	30
	3.4 Blossoms	31
	3.5 Barycentric Coordinates in the Plane	34
	3.6 Tessellations	37
	3.7 Triangulations	39
	3.8 Problems	41
Chapter 4	The de Casteljaou Algorithm	43
	4.1 Parabolas	43
	4.2 The de Casteljaou Algorithm	45
	4.3 Some Properties of Bézier Curves	47

4.4	The Blossom	50
4.5	Implementation	53
4.6	Problems	54
Chapter 5	The Bernstein Form of a Bézier Curve	57
5.1	Bernstein Polynomials	57
5.2	Properties of Bézier Curves	60
5.3	The Derivatives of a Bézier Curve	62
5.4	Domain Changes and Subdivision	68
5.5	Composite Bézier Curves	71
5.6	Blossom and Polar	73
5.7	The Matrix Form of a Bézier Curve	74
5.8	Implementation	75
5.9	Problems	78
Chapter 6	Bézier Curve Topics	81
6.1	Degree Elevation	81
6.2	Repeated Degree Elevation	83
6.3	The Variation Diminishing Property	84
6.4	Degree Reduction	85
6.5	Nonparametric Curves	86
6.6	Cross Plots	87
6.7	Integrals	88
6.8	The Bézier Form of a Bézier Curve	89
6.9	The Weierstrass Approximation Theorem	90
6.10	Formulas for Bernstein Polynomials	91
6.11	Implementation	92
6.12	Problems	93
Chapter 7	Polynomial Curve Constructions	95
7.1	Aitken's Algorithm	95
7.2	Lagrange Polynomials	98
7.3	The Vandermonde Approach	99
7.4	Limits of Lagrange Interpolation	101
7.5	Cubic Hermite Interpolation	102
7.6	Quintic Hermite Interpolation	106
7.7	Point-Normal Interpolation	107

7.8	Least Squares Approximation	108
7.9	Smoothing Equations	110
7.10	Designing with Bézier Curves	112
7.11	The Newton Form and Forward Differencing	114
7.12	Implementation	116
7.13	Problems	117
Chapter 8	B-Spline Curves	119
8.1	Motivation	120
8.2	B-Spline Segments	122
8.3	B-Spline Curves	126
8.4	Knot Insertion	130
8.5	Degree Elevation	133
8.6	Greville Abcissae	134
8.7	Smoothness	135
8.8	B-Splines	140
8.9	B-Spline Basics	143
8.10	Implementation	144
8.11	Problems	146
Chapter 9	Constructing Spline Curves	147
9.1	Greville Interpolation	147
9.2	Least Squares Approximation	149
9.3	Modifying B-Spline Curves	153
9.4	C^2 Cubic Spline Interpolation	154
9.5	More End Conditions	157
9.6	Finding a Knot Sequence	161
9.7	The Minimum Property	167
9.8	C^1 Piecewise Cubic Interpolation	169
9.9	Implementation	174
9.10	Problems	176
Chapter 10	W. Boehm: Differential Geometry I	179
10.1	Parametric Curves and Arc Length	179
10.2	The Frenet Frame	181
10.3	Moving the Frame	182
10.4	The Osculating Circle	184

	10.5	Nonparametric Curves	187
	10.6	Composite Curves	188
Chapter 11		Geometric Continuity	191
	11.1	Motivation	191
	11.2	The Direct Formulation	192
	11.3	The γ , ν , and β Formulations	194
	11.4	G^2 Cubic Splines	195
	11.5	Interpolating G^2 Cubic Splines	199
	11.6	Higher-Order Geometric Continuity	200
	11.7	Implementation	203
	11.8	Problems	203
Chapter 12		Conic Sections	205
	12.1	Projective Maps of the Real Line	205
	12.2	Conics as Rational Quadratics	209
	12.3	A de Casteljaou Algorithm	214
	12.4	Derivatives	215
	12.5	The Implicit Form	216
	12.6	Two Classic Problems	219
	12.7	Classification	220
	12.8	Control Vectors	223
	12.9	Implementation	224
	12.10	Problems	225
Chapter 13		Rational Bézier and B-Spline Curves	227
	13.1	Rational Bézier Curves	227
	13.2	The de Casteljaou Algorithm	230
	13.3	Derivatives	233
	13.4	Osculatory Interpolation	234
	13.5	Reparametrization and Degree Elevation	235
	13.6	Control Vectors	238
	13.7	Rational Cubic B-Spline Curves	238
	13.8	Interpolation with Rational Cubics	240
	13.9	Rational B-Splines of Arbitrary Degree	241
	13.10	Implementation	242
	13.11	Problems	243

Chapter 14	Tensor Product Patches	245
14.1	Bilinear Interpolation	245
14.2	The Direct de Casteljau Algorithm	247
14.3	The Tensor Product Approach	250
14.4	Properties	253
14.5	Degree Elevation	254
14.6	Derivatives	255
14.7	Blossoms	258
14.8	Curves on a Surface	260
14.9	Normal Vectors	261
14.10	Twists	264
14.11	The Matrix Form of a Bézier Patch	265
14.12	Nonparametric Patches	266
14.13	Problems	267
Chapter 15	Constructing Polynomial Patches	269
15.1	Ruled Surfaces	269
15.2	Coons Patches	270
15.3	Translational Surfaces	272
15.4	Tensor Product Interpolation	274
15.5	Bicubic Hermite Patches	276
15.6	Least Squares	278
15.7	Finding Parameter Values	281
15.8	Shape Equations	282
15.9	A Problem with Unstructured Data	282
15.10	Implementation	283
15.11	Problems	284
Chapter 16	Composite Surfaces	285
16.1	Smoothness and Subdivision	285
16.2	Tensor Product B-Spline Surfaces	288
16.3	Twist Estimation	290
16.4	Bicubic Spline Interpolation	293
16.5	Finding Knot Sequences	295
16.6	Rational Bézier and B-Spline Surfaces	297
16.7	Surfaces of Revolution	299

16.8	Volume Deformations	301
16.9	CONS and Trimmed Surfaces	304
16.10	Implementation	306
16.11	Problems	308
Chapter 17	Bézier Triangles	309
17.1	The de Casteljau Algorithm	309
17.2	Triangular Blossoms	313
17.3	Bernstein Polynomials	315
17.4	Derivatives	316
17.5	Subdivision	320
17.6	Differentiability	323
17.7	Degree Elevation	326
17.8	Nonparametric Patches	326
17.9	The Multivariate Case	328
17.10	S-Patches	330
17.11	Implementation	331
17.12	Problems	332
Chapter 18	Practical Aspects of Bézier Triangles	335
18.1	Rational Bézier Triangles	335
18.2	Quadrics	337
18.3	Interpolation	341
18.4	Cubic and Quintic Interpolants	341
18.5	The Clough–Tocher Interpolant	343
18.6	The Powell–Sabin Interpolant	345
18.7	Least Squares	346
18.8	Problems	347
Chapter 19	W. Boehm: Differential Geometry II	349
19.1	Parametric Surfaces and Arc Element	349
19.2	The Local Frame	352
19.3	The Curvature of a Surface Curve	352
19.4	Meusnier’s Theorem	354
19.5	Lines of Curvature	355
19.6	Gaussian and Mean Curvature	357
19.7	Euler’s Theorem	358

19.8	Dupin's Indicatrix	359
19.9	Asymptotic Lines and Conjugate Directions	360
19.10	Ruled Surfaces and Developables	361
19.11	Nonparametric Surfaces	363
19.12	Composite Surfaces	364
Chapter 20	Geometric Continuity for Surfaces	367
20.1	Introduction	367
20.2	Triangle-Triangle	368
20.3	Rectangle-Rectangle	372
20.4	Rectangle-Triangle	373
20.5	"Filling in" Rectangular Patches	374
20.6	"Filling in" Triangular Patches	375
20.7	Theoretical Aspects	375
20.8	Problems	376
Chapter 21	Surfaces with Arbitrary Topology	377
21.1	Recursive Subdivision Curves	377
21.2	Doo–Sabin Surfaces	380
21.3	Catmull–Clark Subdivision	383
21.4	Midpoint Subdivision	386
21.5	Loop Subdivision	387
21.6	$\sqrt{3}$ Subdivision	389
21.7	Interpolating Subdivision Surfaces	389
21.8	Surface Splines	392
21.9	Triangular Meshes	394
21.10	Decimation	395
21.11	Problems	398
Chapter 22	Coons Patches	399
22.1	Coons Patches: Bilinearly Blended	400
22.2	Coons Patches: Partially Bicubically Blended	402
22.3	Coons Patches: Bicubically Blended	404
22.4	Piecewise Coons Surfaces	406
22.5	Two Properties	407
22.6	Compatibility	408
22.7	Gordon Surfaces	410

22.8	Boolean Sums	412
22.9	Triangular Coons Patches	414
22.10	Problems	417
Chapter 23	Shape	419
23.1	Use of Curvature Plots	419
23.2	Curve and Surface Smoothing	421
23.3	Surface Interrogation	425
23.4	Implementation	427
23.5	Problems	429
Chapter 24	Evaluation of Some Methods	431
24.1	Bézier Curves or B-Spline Curves?	431
24.2	Spline Curves or B-Spline Curves?	431
24.3	The Monomial or the Bézier Form?	432
24.4	The B-Spline or the Hermite Form?	435
24.5	Triangular or Rectangular Patches?	435
Appendix A	Quick Reference of Curve and Surface Terms	437
Appendix B	List of Programs	445
Appendix C	Notation	447
	References	449
	Index	491