

Contents

| | |
|----------------------------------------------------|------------|
| Preface | ix |
| 1 Introduction | 1 |
| 1.1 Stochastic processes | 1 |
| 1.2 The Markov property | 2 |
| 1.3 Some examples | 6 |
| 1.4 Transition probabilities | 13 |
| 1.5 The strong Markov property | 19 |
| 1.6 Exercises | 21 |
| 2 Discrete-time Markov chains | 25 |
| 2.1 First passage times | 25 |
| 2.2 Classification of states | 30 |
| 2.3 Recurrent Markov chains | 38 |
| 2.4 Finite Markov chains | 51 |
| 2.5 Time-reversible Markov chains | 58 |
| 2.6 The rate of convergence to stationarity | 64 |
| 2.7 Absorbing Markov chains and their applications | 75 |
| 2.8 Lossy Markov chains | 84 |
| 2.9 Exercises | 95 |
| 3 Monotone Markov chains | 101 |
| 3.1 Preliminaries | 102 |
| 3.2 Distribution classes of interest | 112 |
| 3.3 Stochastic ordering relations | 121 |
| 3.4 Monotone Markov chains | 129 |
| 3.5 Unimodality of transition probabilities | 135 |
| 3.6 First-passage-time distributions | 142 |
| 3.7 Bounds for quasi-stationary distributions | 147 |

| | |
|------------------------------------------------------|------------|
| 3.8 Renewal processes in discrete time | 150 |
| 3.9 Comparability of Markov chains | 158 |
| 3.10 Exercises | 163 |
| 4 Continuous-time Markov chains | 167 |
| 4.1 Transition probability functions | 167 |
| 4.2 Finite Markov chains in continuous time | 173 |
| 4.3 Denumerable Markov chains in continuous time | 183 |
| 4.4 Uniformization | 195 |
| 4.5 More on finite Markov chains | 200 |
| 4.6 Absorbing Markov chains in continuous time | 208 |
| 4.7 Calculation of transition probability functions | 216 |
| 4.8 Stochastic monotonicity | 224 |
| 4.9 Semi-Markov processes | 229 |
| 4.10 Exercises | 235 |
| 5 Birth–death processes | 243 |
| 5.1 Boundary classification | 244 |
| 5.2 Birth–death polynomials | 248 |
| 5.3 Finite birth–death processes | 253 |
| 5.4 The Karlin–McGregor representation theorem | 261 |
| 5.5 Asymptotics of birth–death polynomials | 266 |
| 5.6 Quasi-stationary distributions | 271 |
| 5.7 The decay parameter | 281 |
| 5.8 The M/M/1 queue | 287 |
| 5.9 Exercises | 290 |
| A Review of matrix theory | 295 |
| A.1 Nonnegative matrices | 295 |
| A.2 ML-matrices | 298 |
| A.3 Infinite matrices | 300 |
| B Generating functions and Laplace transforms | 303 |
| B.1 Generating functions | 303 |
| B.2 Laplace transforms | 307 |
| C Total positivity | 313 |
| C.1 TP _r functions | 313 |
| C.2 The variation-diminishing property | 316 |
| References | 319 |
| Symbols | 329 |

| | |
|----------------------|------------|
| <i>Contents</i> | vii |
| Author index | 331 |
| Subject index | 334 |