

Contents

Preface to the Second Edition	xi
1. Some Algebraic Geometry	1
1.1. The Zariski topology	1
1.2. Irreducibility of topological spaces	2
1.3. Affine algebras	4
1.4. Regular functions, ringed spaces	6
1.5. Products	10
1.6. Prevarieties and varieties	11
1.7. Projective varieties	14
1.8. Dimension	16
1.9. Some results on morphisms	17
Notes	20
2. Linear Algebraic Groups, First Properties	21
2.1. Algebraic groups	21
2.2. Some basic results	25
2.3. G -spaces	28
2.4. Jordan decomposition	31
2.5. Recovering a group from its representations	37
Notes	41
3. Commutative Algebraic Groups	42
3.1. Structure of commutative algebraic groups	42
3.2. Diagonalizable groups and tori	43

3.3. Additive functions	49
3.4. Elementary unipotent groups	51
Notes	56
4. Derivations, Differentials, Lie Algebras	57
4.1. Derivations and tangent spaces	57
4.2. Differentials, separability	60
4.3. Simple points	66
4.4. The Lie algebra of a linear algebraic group	69
Notes	77
5. Topological Properties of Morphisms, Applications	78
5.1. Topological properties of morphisms	78
5.2. Finite morphisms, normality	82
5.3. Homogeneous spaces	86
5.4. Semi-simple automorphisms	88
5.5. Quotients	91
Notes	97
6. Parabolic Subgroups, Borel Subgroups, Solvable Groups	98
6.1. Complete varieties	98
6.2. Parabolic subgroups and Borel subgroups	101
6.3. Connected solvable groups	104
6.4. Maximal tori, further properties of Borel groups	108
Notes	113

7. Weyl Group, Roots, Root Datum	114
7.1. The Weyl group	114
7.2. Semi-simple groups of rank one	117
7.3. Reductive groups of semi-simple rank one	120
7.4. Root data	124
7.5. Two roots	128
7.6. The unipotent radical	130
Notes	131
8. Reductive Groups.....	132
8.1. Structural properties of a reductive group	132
8.2. Borel subgroups and systems of positive roots	137
8.3. The Bruhat decomposition	142
8.4. Parabolic subgroups	146
8.5. Geometric questions related to the Bruhat decomposition	149
Notes	153
9. The Isomorphism Theorem.....	154
9.1. Two dimensional root systems	154
9.2. The structure constants	156
9.3. The elements n_α	162
9.4. A presentation of G	164
9.5. Uniqueness of structure constants	168
9.6. The isomorphism theorem	170
Notes	174

10. The Existence Theorem	175
10.1. Statement of the theorem, reduction	175
10.2. Simply laced root systems	177
10.3. Automorphisms, end of the proof of 10.1.1	181
Notes	184
11. More Algebraic Geometry	185
11.1. F -structures on vector spaces	185
11.2. F -varieties: density, criteria for ground fields	191
11.3. Forms	196
11.4. Restriction of the ground field	198
Notes	207
12. F-groups: General Results	208
12.1. Field of definition of subgroups	208
12.2. Complements on quotients	212
12.3. Galois cohomology	216
12.4. Restriction of the ground field	220
Notes	222
13. F-tori	223
13.1. Diagonalizable groups over F	223
13.2. F -tori	225
13.3. Tori in F -groups	227
13.4. The groups $P(\lambda)$	233
Notes	236

14. Solvable F-groups	237
14.1. Generalities	237
14.2. Action of \mathbf{G}_a on an affine variety, applications	239
14.3. F -split solvable groups	243
14.4. Structural properties of solvable groups	248
Notes	251
15. F-reductive Groups	252
15.1. Pseudo-parabolic F -subgroups	252
15.2. A fixed point theorem	254
15.3. The root datum of an F -reductive group	256
15.4. The groups $U_{(a)}$	262
15.5. The index	265
Notes	268
16. Reductive F-groups	269
16.1. Parabolic subgroups	269
16.2. Indexed root data	271
16.3. F -split groups	274
16.4. The isomorphism theorem	278
16.5. Existence	281
Notes	284
17. Classification	285
17.1. Type A_{n-1}	285

17.2. Types B_n and C_n	289
17.3. Type D_n	293
17.4. Exceptional groups, type G_2	300
17.5. Indices for types F_4 and E_8	302
17.6. Descriptions for type F_4	305
17.7. Type E_6	310
17.8. Type E_7	312
17.9. Trialitarian type D_4	315
17.10. Special fields	317
Notes	319
Table of Indices	320
Bibliography	323
Index	331