

# Contents

<i>Preface</i> . . . . .	v
<i>Notation and Symbols</i> . . . . .	ix
<i>Introduction</i> . . . . .	1
<i>I Limit Theorems for Stopped Random Walks</i> . . . . .	8
1 Introduction . . . . .	8
2 a.s. Convergence and Convergence in Probability . . . . .	10
3 Anscombe's Theorem . . . . .	15
4 Moment Convergence in the Strong Law and the Central Limit Theorem . . . . .	17
5 Moment Inequalities . . . . .	20
6 Uniform Integrability . . . . .	28
7 Moment Convergence . . . . .	36
8 The Stopping Summand . . . . .	39
9 The Law of the Iterated Logarithm . . . . .	41
10 Complete Convergence and Convergence Rates . . . . .	42
11 Problems . . . . .	44
<i>II Renewal Processes and Random Walks</i> . . . . .	46
1 Introduction . . . . .	46
2 Renewal Processes; Introductory Examples . . . . .	47
3 Renewal Processes; Definition and General Facts . . . . .	48
4 Renewal Theorems . . . . .	51
5 Limit Theorems . . . . .	54
6 The Residual Lifetime . . . . .	58
7 Further Results . . . . .	60
8 Random Walks; Introduction and Classifications . . . . .	62
9 Ladder Variables . . . . .	65
10 The Maximum and the Minimum of a Random Walk . . . . .	67
11 Representation Formulas for the Maximum . . . . .	68
12 Limit Theorems for the Maximum . . . . .	70

<i>III</i>	<i>Renewal Theory for Random Walks with Positive Drift</i> . . . . .	74
1	Introduction . . . . .	74
2	Ladder Variables . . . . .	77
3	Finiteness of Moments. . . . .	78
4	The Strong Law of Large Numbers . . . . .	83
5	The Central Limit Theorem . . . . .	85
6	Renewal Theorems. . . . .	87
7	Uniform Integrability. . . . .	90
8	Moment Convergence . . . . .	92
9	Further Results on $Ev(t)$ and $\text{Var } v(t)$ . . . . .	94
10	The Overshoot . . . . .	97
11	The Law of the Iterated Logarithm . . . . .	102
12	Complete Convergence and Convergence Rates. . . . .	103
13	Applications to the Simple Random Walk . . . . .	104
14	Extensions to the Non-I.I.D. Case . . . . .	106
15	Problems . . . . .	107
<i>IV</i>	<i>Generalizations and Extensions</i> . . . . .	108
1	Introduction . . . . .	108
2	A Stopped Two-Dimensional Random Walk . . . . .	109
3	Some Applications . . . . .	118
4	The Maximum of a Random Walk with Positive Drift . . . . .	128
5	First Passage Times Across General Boundaries . . . . .	133
<i>V</i>	<i>Functional Limit Theorems</i> . . . . .	147
1	Introduction . . . . .	147
2	An Anscombe–Donsker Invariance Principle. . . . .	147
3	First Passage Times for Random Walks with Positive Drift. . . . .	151
4	A Stopped Two-Dimensional Random Walk . . . . .	157
5	The Maximum of a Random Walk with Positive Drift . . . . .	159
6	First Passage Times Across General Boundaries . . . . .	159
7	The Law of the Iterated Logarithm . . . . .	161
8	Further Results. . . . .	164
	<i>Appendix A. Some Facts from Probability Theory</i> . . . . .	165
1	Convergence of Moments. Uniform Integrability. . . . .	165
2	Moment Inequalities for Martingales . . . . .	167
3	Convergence of Probability Measures. . . . .	171
4	Strong Invariance Principles. . . . .	177
5	Problems. . . . .	178
	<i>Appendix B. Some Facts about Regularly Varying Functions</i> . . . . .	180
1	Introduction and Definitions . . . . .	180
2	Some Results. . . . .	181
	<i>Bibliography</i> . . . . .	183
	<i>Index</i> . . . . .	193