

Contents

Preface	vii	
Notation	xiii	
I	Deterministic Optimal Control	1
I.1	Introduction	1
I.2	Examples	2
I.3	Finite time horizon problems	5
I.4	Dynamic programming principle	9
I.5	Dynamic programming equation	11
I.6	Dynamic programming and Pontryagin's principle	18
I.7	Discounted cost with infinite horizon	23
I.8	Calculus of variations I	32
I.9	Calculus of variations II	37
I.10	Generalized solutions to Hamilton-Jacobi equations	43
I.11	Historical remarks	51
II	Viscosity Solutions	53
II.1	Introduction	53
II.2	Examples	56
II.3	An abstract dynamic programming principle	58
II.4	Definition	64
II.5	Dynamic programming and viscosity property	68
II.6	Properties of viscosity solutions	70
II.7	Deterministic optimal control and viscosity solutions	75
II.8	Viscosity solutions: First-order case	79
II.9	Uniqueness: First-order case	85
II.10	Continuity of the value function	95
II.11	Discounted cost with infinite horizon	99
II.12	State constraint	101
II.13	Discussion of boundary conditions	106
II.14	Uniqueness: First-order case	108
II.15	Pontryagin's maximum principle	109
II.16	Unbounded control set U	111
II.17	Historical remarks	122

III	Optimal Control of Markov Processes: Classical Solutions	125
III.1	Introduction	125
III.2	Markov processes and their evolution operators	126
III.3	Autonomous (time-homogeneous) Markov processes	129
III.4	Classes of Markov processes	131
III.5	Markov diffusion processes on \mathbb{R}^n ; stochastic differential equations	133
III.6	Controlled Markov processes	136
III.7	Dynamic programming: Formal description	137
III.8	A Verification Theorem; finite time horizon	140
III.9	Infinite time horizon	145
III.10	Viscosity solutions	151
III.11	Historical remarks	155
IV	Controlled Markov Diffusions in \mathbb{R}^n	157
IV.1	Introduction	157
IV.2	Finite time horizon problem	158
IV.3	Hamilton-Jacobi-Bellman PDE	161
IV.4	Uniformly parabolic case	167
IV.5	Infinite time horizon	171
IV.6	Fixed finite time horizon problem: Preliminary estimates	177
IV.7	Dynamic programming principle	182
IV.8	Estimates for first-order difference quotients	189
IV.9	Estimates for second-order difference quotients	193
IV.10	Generalized subsolutions and solutions	197
IV.11	Stochastic calculus of variations	205
IV.12	Historical remarks	210
V	Viscosity Solutions: Second-Order Case	213
V.1	Introduction	213
V.2	Dynamic programming principle	214
V.3	Viscosity property	219
V.4	An equivalent formulation	223
V.5	Jensen's maximum principle	227
V.6	Ishii's lemma	237
V.7	Semiconvex, concave approximations	240
V.8	Comparison	243
V.9	Viscosity solutions in Q_0	247
V.10	Historical remarks	251

VI	Logarithmic Transformations	253
VI.1	Introduction	253
VI.2	Nondegenerate diffusions in \mathbb{R}^n	254
VI.3	Locally optimal Markov policies	259
VI.4	Conditioned Markov diffusions	262
VI.5	The exit problem	265
VI.6	Small noise limits I	267
VI.7	Small noise limits II: Asymptotic series	271
VI.8	Logarithmic transformations for Markov processes	275
VI.9	Historical remarks	278
VII	Singular Perturbations	281
VII.1	Introduction	281
VII.2	Examples	283
VII.3	Barles and Perthame procedure	285
VII.4	Discontinuous viscosity solutions	287
VII.5	Terminal condition	289
VII.6	Boundary condition	291
VII.7	Convergence	292
VII.8	Comparison	293
VII.9	Vanishing viscosity	301
VII.10	Large deviations for exit probabilities	303
VII.11	Historical remarks	313
VIII	Singular Stochastic Control	315
VIII.1	Introduction	315
VIII.2	Formal discussion	316
VIII.3	Singular stochastic control	318
VIII.4	Verification theorem	321
VIII.5	Viscosity solutions	333
VIII.6	Portfolio selection with transaction costs	339
VIII.7	Optimal investment/transaction policy	342
VIII.8	Finite fuel problem	359
VIII.9	Historical remarks	362
IX	Finite-Difference Numerical Approximations	363
IX.1	Introduction	363
IX.2	Controlled discrete-time Markov chains	364
IX.3	Finite-difference approximations to HJB equations	366
IX.4	Convergence of finite-difference approximations I	374
IX.5	Convergence of finite-difference approximations II	379
IX.6	Historical remarks	388

Appendix A Duality Relationships	389
Appendix B Dynkin's Formula for Random Evolutions with Markov Chain Parameters	391
Appendix C Extension of Lipschitz Continuous Functions; Smoothing	393
Appendix D Stochastic Differential Equations: Random Coefficients	397
Appendix E A Result of Alexandrov	403
References	409
Index	425