

CONTENTS

PREFACE	v
INTERDEPENDENCE GUIDE	vii
CONTENTS	viii
CHAPTER 0. PRELIMINARIES	1
1. Notation	1
2. Martingales	3
3. The monotone class theorem	4
4. Topological spaces and groups	6
CHAPTER 1. TRANSITION PROBABILITIES. MARKOV CHAINS	8
1. Kernels. Transition probabilities	8
2. Homogeneous Markov chains	13
3. Stopping times. Strong Markov property	21
4. Random walks on groups and homogeneous spaces	26
5. Analytical properties of integral kernels	31
CHAPTER 2. POTENTIAL THEORY	37
1. Superharmonic functions and the maximum principle	37
2. Reduced functions	44
3. Invariant and excessive measures	52
4. Resolvents	60
CHAPTER 3. TRANSIENCE AND RECURRENCE	65
1. Discrete Markov chains	65
2. Irreducible chains and Harris chains	71
3. Invariant events. Transient and recurrent sets	82
4. Topological recurrence of random walks	87
5. Recurrence criteria for random walks and applications	95

CHAPTER 4. POINTWISE ERGODIC THEORY	106
1. Preliminaries	106
2. Maximal ergodic lemma. Hopf's decomposition	110
3. The Chacon–Ornstein theorem for conservative contractions and applications	117
4. Brunel's lemma and the general Chacon–Ornstein theorem	126
CHAPTER 5. TRANSIENT RANDOM WALKS. RENEWAL THEORY	133
1. The theorem of Choquet and Deny	133
2. General lemmas	137
3. The renewal theorem for the groups \mathbf{R} and \mathbf{Z}	142
4. The renewal theorem	150
5. Refinements and applications	153
CHAPTER 6. ERGODIC THEORY OF HARRIS CHAINS	159
1. Cyclic classes	159
2. Orey's theorem	164
3. Quasi-compact transition probabilities and strong ergodic theorem	173
4. Special functions	182
5. Potential kernels	187
6. The ratio-limit theorem	197
CHAPTER 7. MARTIN BOUNDARY	203
1. Regular functions	203
2. Convergence to the boundary	211
3. Integral representation of harmonic functions	222
CHAPTER 8. POTENTIAL THEORY FOR HARRIS CHAINS	231
1. Harris chains and duality	231
2. Equilibrium, balayage and maximum principle	236
3. Normal chains	240
4. Feller chains and recurrent boundary theory	247
CHAPTER 9. RECURRENT RANDOM WALKS	256
1. Preliminaries	256
2. Normality and potential kernels	268
3. Martin boundary	278
4. Renewal theory	285

CHAPTER 10. CONSTRUCTION OF MARKOV CHAINS AND RESOLVENTS	294
1. Preliminaries and bounded kernels	294
2. The reinforced principle. Construction of transient Markov chains . .	300
3. The semi-complete maximum principle	308
NOTES AND COMMENTS	316
REFERENCES	325
INDEX	335