Contents

| Introduction | xii |
|--|----------------|
| Part I: Control on a Finite Time Interval | 1 |
| Chaper 1. Finite and Denumerable Models | 3 |
| \$1. Deterministic Controlled Processes | 3 8 15 |
| Strategies | 18 19 |
| Problem for the Derived Model | 20 |
| Optimal Strategies | 23 25 27 |
| §10. The Bus, Streetcar, or Walk Problem | 27 33 |
| §12. Countable Models: Optimality Equations and ε-Optimal Strategies | 35 39 |
| Chapter 2. Semicontinuous Models | 44 |
| \$1. On the Concept of Measurability | 44 45 |
| in the Study of Finite and Countable Models? | 48 50 |
| \$5. Optimality Equations and Simple Optimal Strategies | 53 57 |
| Consumption | 61 |

| | Contents |
|--|----------|
|--|----------|

X

| §8. The Water Regulation Problem | 64 66 |
|--|--|
| §10. The Problem of Allocation of a Resource Among Consumption and Several Productive Sectors | 74 76 |
| Chapter 3. General (Borel) Models | 79 |
| \$1. Introduction. The Main Results | 79 83 87 87 91 |
| §6. Universal Measurability of the Value of the Model and Almost-Surely (a.s.) ε-Optimal Strategies | 92 94 96 98 |
| Part II: Control on an Infinite Time Interval | 101 |
| Chapter 4. Discrete Models | 103 |
| \$1. Passage to an Infinite Interval of Control \$2. Summable Models . \$3. The Fundamental Equation \$4. Uniformly ε-Optimal Strategies \$5. Optimality Equations \$6. An Expression for the Value of a Model \$7. Simple ε-Optimal Strategies \$8. Sufficiency of Markov and Simple Strategies | 103 103 105 107 110 112 114 118 |
| Chapter 5. Borel Models | 121 |
| \$1. The Main Results | 121 122 126 128 |
| of a.s. ε-Optimal Strategies | 129 130 |
| Chapter 6. Homogeneous Models | 133 |
| §1. Introduction | 133 134 137 |

| хi |
|----|
| |

| §4. The Bus, Streetcar, or Walk Problem | 140 |
|--|------------|
| | 143 |
| | 147 |
| | 150 |
| | 153 |
| §9. Allocation of a Resource Between Production and | |
| | 155 |
| | 159 |
| §11. Allocation of a Resource Among Consumption and Several | |
| | 160 |
| | 162 |
| Chapter 7. Maximization of the Average Reward Per Unit Time | 165 |
| §1. Introduction. Canonical Strategies | 165 |
| _ i | 167 |
| | 169 |
| | 172 |
| | 173 |
| | 174 |
| §7. Increase of the Discounted Reward with the Howard | 1/4 |
| ~ ~ | 176 |
| - · · · · · · · · · · · · · · · · · · · | 176 178 |
| §8. Extension to Infinite Models | 1/0 |
| | 104 |
| | 184 |
| | 186 |
| | 188 |
| 912. The Stabilization Problem | 193 |
| §13. Models with Finitely Many States and Infinite Action Sets | 195 |
| | |
| Part III: Some Applications | 199 |
| Chapter 8. Models with Incomplete Information | 201 |
| §1. Description of the Model | 201 |
| §2. Reduction to a Model with Complete Information. The | |
| | 202 |
| V | 209 |
| §4. Reduction to a Model with Complete Information. The | |
| | 214 |
| §5. The Stabilization Problem | 217 |
| Chapter 9. Concave Models. Models of Economic Development | 222 |
| | |
| | 222 |
| §2. Concave Models | 223 |
| | |

| xii | | | | | Con | tents |
|--|----|---|---|---|-----|---------------------------------|
| §3. The Spaces \mathscr{L} | | | | | | 226 230 232 |
| Appendix 1: Borel Spaces | | | | | | 239 |
| §1. Introduction | | • | | | • | 239 240 |
| Uncountable Borel Space | • | • | • | • | • | 241 |
| Dyadic Sequences | | | | • | • | 244 |
| Appendix 2: Analytic Sets | | | | | | 246 |
| \$1. Introduction | | | | | · | 246 247 248 250 251 |
| Appendix 3: Theorems on Measurable Selection | | | | | | 254 |
| \$1. The Lemma of Yankov \$2. The Theorem of Blackwell and Ryll-Nardzewski \$3. Example of a Correspondence Not Admitting a Measurab Selection | | | | | | 254 255 256 |
| Appendix 4: Conditional Distributions | | | | | | 258 |
| \$1. Introduction | | | | | | 258 259 261 262 |
| Appendix 5: Some Lemmas on Measurability | | | | | | 265 |
| §1. The Lemma on Multiplicative Systems§2. Measurable Structure in the Space of Probability Measure | es | • | | | | 265 266 |
| Historical-Bibliographical Notes | | | | | | 267 |
| Bibliography | | | | | | 275 |
| Index | | | | | | 287 |