Table of Contents

Chapter I

Basic Notions and Illustrations

0. Summary 1
1. Markov Processes and Transition Probability Functions . . . 1
2. Markov Chains 7
3. Independent Random Variables 17
4. Some Continuous Parameter Markov Processes 28
5. Random Walks on Countable Commutative Groups 31
Notes 40

Chapter II

Remarks on Some Applications

0. Summary 45
1. A Model in Statistical Mechanics 45
2. Some Models in Learning Theory 51
3. A Resource Flow Model 54
Notes 56

Chapter III

Functions of Markov Processes

0. Summary 57
1. Collapsing of States and the Chapman-Kolmogorov Equation 57
2. Markovian Functions of Markov Processes 65
3. Functions of Finite State Markov Chains 72
Notes 77

Chapter IV

Ergodic and Prediction Problems

0. Summary 79
1. A Markov Process Restricted to a Set A 80
2. An L^1 Ergodic Theorem 83
3. Transition Operators and Invariant Measures on a Topological Space 98
4. Asymptotic Behavior of Powers of a Transition Probability Operator 105
Notes 116

Chapter V

Random Walks and Convolution on Groups and Semigroups

0. Summary 119
1. A Problem of P. Lévy 119
2. Limit Theorems and the Convolution Operation 125
3. Idempotent Measures as Limiting Distributions 131
4. The Structure of Compact Semigroups 139
5. Convergent Convolution Sequences 149
Notes 158

Chapter VI

Nonlinear Representations in Terms of Independent Random Variables

0. Summary 161
1. The Linear Prediction Problem for Stationary Sequences . . . 161
2. A Nonlinear Prediction Problem 163
3. Questions for Markov Processes 167
4. Finite State Markov Chains 170
5. Real-Valued Markov Processes 174
Notes 180

Chapter VII

Mixing and the Central Limit Theorem

0. Summary 183
1. Independence 183

2. Uniform Ergodicity, Strong Mixing and the Central Limit Problem 193
3. An Operator Formulation of Strong Mixing and Uniform Ergodicity 199
4. L^p Norm Conditions and a Central Limit Theorem 206
Notes 218

Appendix 1. Probability Theory 220
Appendix 2. Topological Spaces 228
Appendix 3. The Kolmogorov Extension Theorem 234
Appendix 4. Spaces and Operators 240
Appendix 5. Topological Groups 248

Bibliography 254

Postscript 259

Author Index 261

Subject Index 263

Notation 267