

Contents

Preface	ix
About the Authors	xiii
1 Introduction and Overview	1
1.1 Lagrangian and Hamiltonian Formalisms	1
1.2 The Rigid Body	6
1.3 Lie–Poisson Brackets, Poisson Manifolds, Momentum Maps	9
1.4 The Heavy Top	16
1.5 Incompressible Fluids	18
1.6 The Maxwell–Vlasov System	22
1.7 Nonlinear Stability	29
1.8 Bifurcation	43
1.9 The Poincaré–Melnikov Method	47
1.10 Resonances, Geometric Phases, and Control	50
2 Hamiltonian Systems on Linear Symplectic Spaces	61
2.1 Introduction	61
2.2 Symplectic Forms on Vector Spaces	66
2.3 Canonical Transformations, or Symplectic Maps	69
2.4 The General Hamilton Equations	74
2.5 When Are Equations Hamiltonian?	77
2.6 Hamiltonian Flows	80
2.7 Poisson Brackets	82

2.8	A Particle in a Rotating Hoop	87
2.9	The Poincaré–Melnikov Method	94
3	An Introduction to Infinite-Dimensional Systems	105
3.1	Lagrange’s and Hamilton’s Equations for Field Theory . .	105
3.2	Examples: Hamilton’s Equations	107
3.3	Examples: Poisson Brackets and Conserved Quantities . .	115
4	Manifolds, Vector Fields, and Differential Forms	121
4.1	Manifolds	121
4.2	Differential Forms	129
4.3	The Lie Derivative	137
4.4	Stokes’ Theorem	141
5	Hamiltonian Systems on Symplectic Manifolds	147
5.1	Symplectic Manifolds	147
5.2	Symplectic Transformations	150
5.3	Complex Structures and Kähler Manifolds	152
5.4	Hamiltonian Systems	157
5.5	Poisson Brackets on Symplectic Manifolds	160
6	Cotangent Bundles	165
6.1	The Linear Case	165
6.2	The Nonlinear Case	167
6.3	Cotangent Lifts	170
6.4	Lifts of Actions	173
6.5	Generating Functions	174
6.6	Fiber Translations and Magnetic Terms	176
6.7	A Particle in a Magnetic Field	178
7	Lagrangian Mechanics	181
7.1	Hamilton’s Principle of Critical Action	181
7.2	The Legendre Transform	183
7.3	Euler–Lagrange Equations	185
7.4	Hyperregular Lagrangians and Hamiltonians	188
7.5	Geodesics	195
7.6	The Kaluza–Klein Approach to Charged Particles	200
7.7	Motion in a Potential Field	202
7.8	The Lagrange–d’Alembert Principle	205
7.9	The Hamilton–Jacobi Equation	210
8	Variational Principles, Constraints, & Rotating Systems	219
8.1	A Return to Variational Principles	219
8.2	The Geometry of Variational Principles	226
8.3	Constrained Systems	234

8.4	Constrained Motion in a Potential Field	238
8.5	Dirac Constraints	242
8.6	Centrifugal and Coriolis Forces	248
8.7	The Geometric Phase for a Particle in a Hoop	253
8.8	Moving Systems	257
8.9	Routh Reduction	260
9	An Introduction to Lie Groups	265
9.1	Basic Definitions and Properties	267
9.2	Some Classical Lie Groups	283
9.3	Actions of Lie Groups	309
10	Poisson Manifolds	327
10.1	The Definition of Poisson Manifolds	327
10.2	Hamiltonian Vector Fields and Casimir Functions	333
10.3	Properties of Hamiltonian Flows	338
10.4	The Poisson Tensor	340
10.5	Quotients of Poisson Manifolds	349
10.6	The Schouten Bracket	353
10.7	Generalities on Lie–Poisson Structures	360
11	Momentum Maps	365
11.1	Canonical Actions and Their Infinitesimal Generators	365
11.2	Momentum Maps	367
11.3	An Algebraic Definition of the Momentum Map	370
11.4	Conservation of Momentum Maps	372
11.5	Equivariance of Momentum Maps	378
12	Computation and Properties of Momentum Maps	383
12.1	Momentum Maps on Cotangent Bundles	383
12.2	Examples of Momentum Maps	389
12.3	Equivariance and Infinitesimal Equivariance	396
12.4	Equivariant Momentum Maps Are Poisson	403
12.5	Poisson Automorphisms	412
12.6	Momentum Maps and Casimir Functions	413
13	Lie–Poisson and Euler–Poincaré Reduction	417
13.1	The Lie–Poisson Reduction Theorem	417
13.2	Proof of the Lie–Poisson Reduction Theorem for $GL(n)$	420
13.3	Lie–Poisson Reduction Using Momentum Functions	421
13.4	Reduction and Reconstruction of Dynamics	423
13.5	The Euler–Poincaré Equations	432
13.6	The Lagrange–Poincaré Equations	442

14 Coadjoint Orbits	445
14.1 Examples of Coadjoint Orbits	446
14.2 Tangent Vectors to Coadjoint Orbits	453
14.3 The Symplectic Structure on Coadjoint Orbits	455
14.4 The Orbit Bracket via Restriction of the Lie–Poisson Bracket	461
14.5 The Special Linear Group of the Plane	467
14.6 The Euclidean Group of the Plane	469
14.7 The Euclidean Group of Three-Space	474
15 The Free Rigid Body	483
15.1 Material, Spatial, and Body Coordinates	483
15.2 The Lagrangian of the Free Rigid Body	485
15.3 The Lagrangian and Hamiltonian in Body Representation	487
15.4 Kinematics on Lie Groups	491
15.5 Poincot’s Theorem	492
15.6 Euler Angles	495
15.7 The Hamiltonian of the Free Rigid Body	497
15.8 The Analytical Solution of the Free Rigid-Body Problem .	500
15.9 Rigid-Body Stability	505
15.10 Heavy Top Stability	509
15.11 The Rigid Body and the Pendulum	514
References	521
Index	555