

Contents

1	Introduction	1
2	Prerequisites from Quantum Mechanics	3
2.1	Postulates of Quantum Mechanics	4
2.2	State Transformations	14
2.3	Notes	22
2.4	Exercises	22
3	Information and its Measures	25
3.1	Shannon's Approach	26
3.2	Classical Source Coding	28
3.3	von Neumann Entropy	34
3.4	Quantum Relative Entropy	37
3.5	Rényi Entropy	45
3.6	Notes	49
3.7	Exercises	50
4	Entanglement	53
4.1	Bipartite Systems	53
4.2	Dense Coding and Teleportation	63
4.3	Entanglement Measures	67
4.4	Notes	69
4.5	Exercises	70
5	More About Information Quantities	73
5.1	Shannon's Mutual Information	73
5.2	Markov Chains	74
5.3	Entropy of Parted Systems	76
5.4	Strong Subadditivity of the von Neumann Entropy	78
5.5	The Holevo Quantity	79
5.6	The Entropy Exchange	80

5.7	Notes	81
5.8	Exercises	82
6	Quantum Compression	83
6.1	Distances Between States	83
6.2	Reliable Compression	85
6.3	Universality	88
6.4	Notes	90
6.5	Exercises	90
7	Channels and Their Capacity	91
7.1	Information Channels	91
7.2	The Shannon Capacity	92
7.3	Holevo Capacity	95
7.4	Classical-quantum Channels	104
7.5	Entanglement-assisted Capacity	105
7.6	Notes	106
7.7	Exercises	106
8	Hypothesis Testing	109
8.1	The Quantum Stein Lemma	110
8.2	The Quantum Chernoff Bound	116
8.3	Notes	119
8.4	Exercises	120
9	Coarse-grainings	121
9.1	Basic Examples	121
9.2	Conditional Expectations	123
9.3	Commuting Squares	131
9.4	Superadditivity	133
9.5	Sufficiency	133
9.6	Markov States	138
9.7	Notes	141
9.8	Exercises	142
10	State Estimation	143
10.1	Estimation Schemas	143
10.2	Cramér–Rao Inequalities	150
10.3	Quantum Fisher Information	154
10.4	Contrast Functionals	162
10.5	Notes	163
10.6	Exercises	164

11 Appendix: Auxiliary Linear and Convex Analysis	165
11.1 Hilbert Spaces and Their Operators	165
11.2 Positive Operators and Matrices	167
11.3 Functional Calculus for Matrices	170
11.4 Distances	175
11.5 Majorization	177
11.6 Operator Monotone Functions	180
11.7 Positive Mappings	189
11.8 Matrix Algebras	195
11.9 Conjugate Convex Function	198
11.10 Some Trace Inequalities	199
11.11 Notes	200
11.12 Exercises	200
Bibliography	205
Index	211