

Contents

Volume I

	Page
Preface	V
Preface to the English edition	VIII
Introduction	1
§ 1. Modern definition of a Markov process	1
§ 2. Shift operators. Infinitesimal and characteristic operators	2
§ 3. Diffusion processes. Probabilistic solution of differential equations	4
§ 4. Additive functionals	7
§ 5. Superharmonic and harmonic functions	8
§ 6. Transformations of Markov processes connected with additive functionals	10
§ 7. Generalized Brownian motion	13
§ 8. What is the structure of the most general continuous strong Markov process?	15
§ 9. Nonnegative harmonic functions and asymptotic behavior of paths of a Markov process	17

Chapter One

Contraction semigroups of linear operators on Banach spaces

§ 1. Banach spaces	19
§ 2. Contraction semigroups of linear operators and their infinitesimal operators	22
§ 3. Uniqueness theorems	26
§ 4. Construction of a semigroup from an infinitesimal operator	28
§ 5. Relationship between measurability properties and continuity properties of semigroups of operators	33
§ 6. The weak infinitesimal operator	36
§ 7. Excessive elements	43

Chapter Two

Infinitesimal operators of transition functions

§ 1. Transition functions and corresponding semigroups of operators	47
§ 2. Uniqueness theorems	54
§ 3. Examples	62
§ 4. Feller transition functions on compact spaces	70
§ 5. \hat{C} -Functions on semi-compact spaces	74

Chapter Three

Markov processes

§ 1. Definition of a Markov process	77
§ 2. Markov processes and transition functions	85
§ 3. Strong Markov processes	97

Chapter Four

Page

First entrance and exit times and the intrinsic topology in the state space

§ 1. First entrance, contact and exit times.	104
§ 2. The intrinsic topology in the state space	116
§ 3. Continuous functions in the intrinsic topology	120
§ 4. The intrinsic topology for the Wiener process	127

Chapter Five

Characteristic operators of Markov processes.**Differential generators of diffusion processes**

§ 1. General theorems on resolvents and infinitesimal operators of Markov processes	131
§ 2. Absorbing and stable states	135
§ 3. Definition and general properties of characteristic operators	140
§ 4. Characteristic operators of continuous processes	144
§ 5. Diffusion processes and their differential generators	149
§ 6. Construction of a diffusion process from the differential generator	159
§ 7. Characteristic operators in the intrinsic topology	168

Chapter Six

Functionals of Markov processes

§ 1. Basic definitions	172
§ 2. Operation of passage to the limit	177
§ 3. W -functionals	185
§ 4. Approximation of nonnegative, additive functionals by W -functionals	196
§ 5. Mathematical expectations of random variables, connected with additive functionals	201

Chapter Seven

Stochastic integrals

§ 1. Stochastic integrals as functionals of a Wiener random function.	207
§ 2. A theorem on the transformation of integral functionals.	222
§ 3. Stochastic integrals as functionals of a Wiener process	237

Chapter Eight

Nonnegative additive functionals of a Wiener process

§ 1. Integral representation of a W -function	247
§ 2. W -functionals	255
§ 3. S -functionals	269
§ 4. Functionals of one-dimensional Wiener processes	277

Chapter Nine

Transition functions, corresponding to almost multiplicative functionals

§ 1. Definition and examples.	281
§ 2. Construction of a functional from a quasi-transition function	284
§ 3. Properties of trajectories of Markov processes, corresponding to transformations of transition functions	290
§ 4. Transformation of the resolvent and the infinitesimal operator	293

Chapter Ten

Page

Transformations of Markov processes

§ 1. Curtailment of lifetimes and formation of parts of processes	301
§ 2. Stopped processes	302
§ 3. Transformation of the measures \mathbf{P}_x	306
§ 4. (α, ξ) -subprocesses	308
§ 5. Random time change	320
§ 6. Transformation of the state space	325

Chapter Eleven

Stochastic integral equations and diffusion processes

§ 1. Stochastic integral equations for additive functionals of a Wiener random function	334
§ 2. Construction of diffusion processes	343
§ 3. Stopped diffusion processes	354
Index	358
List of symbols	363

Volume II

Chapter Twelve

Excessive, superharmonic and harmonic functions

§ 1. Excessive functions for transition functions	1
§ 2. Excessive functions for Markov processes	4
§ 3. Asymptotic behavior of excessive functions along trajectories of a process	10
§ 4. Superharmonic functions	15
§ 5. Harmonic functions	24

Chapter Thirteen

Harmonic and superharmonic functions associated with strong Feller processes. Probabilistic solution of certain equations

§ 1. Some properties of strong Feller processes	28
§ 2. The Dirichlet problem. Regular points of the boundary	32
§ 3. Harmonic and superharmonic functions associated with diffusion processes	40
§ 4. Solutions of the equation $\mathcal{A}f - Vf = -g$	46
§ 5. Parts of a diffusion process and Green's functions	53

Chapter Fourteen

The multi-dimensional Wiener process and its transformations

§ 1. Harmonic and superharmonic functions related to the Wiener process	63
§ 2. The mapping Ψ	73
§ 3. Additive functionals and Green's functions	84
§ 4. Brownian motion with killing measure μ and speed measure ν	96
§ 5. q -subprocesses	111
§ 6. Brownian motion with drift	113

Chapter Fifteen

Page

Continuous strong Markov processes on a closed interval

§ 1. General properties of one-dimensional continuous strong Markov processes	119
§ 2. Characteristics of regular processes	129
§ 3. Computation of the characteristic and infinitesimal operators	137
§ 4. Superharmonic and harmonic functions connected with regular one-dimensional processes	146

Chapter Sixteen

Continuous strong Markov processes on an open interval

§ 1. Harmonic functions and behavior of trajectories	148
§ 2. S-functions and character of the motion along a trajectory	155
§ 3. Infinitesimal operators	162

Chapter Seventeen

Construction of one-dimensional continuous strong Markov processes

§ 1. Transformations of the state space. Canonical coordinate	168
§ 2. Construction of regular continuous strong Markov processes on an open interval	176
§ 3. Construction of regular continuous strong Markov processes on a closed interval	187
§ 4. Computation of the harmonic functions and the resolvents for regular processes	193

Appendix

§ 1. Measurable spaces and measurable transformations	201
§ 2. Measures and integrals	204
§ 3. Probability spaces	210
§ 4. Martingales	214
§ 5. Topological measurable spaces	218
§ 6. Some theorems on partial differential equations	225
§ 7. Measures and countably additive set functions on the line and corresponding point functions	231
§ 8. Convex functions	238
Historical-bibliographical note	240
Bibliography	258
Index	267
List of symbols	272