

CONTENTS

Preface	v
CHAPTER 1: SEMINORMED AND NORMED LINEAR SPACES	1
1.0. Introduction	1
1.1. Basic Definitions	1
1.2. Examples of Seminormed Linear Spaces	6
1.3. Finite-Dimensional Normed Linear Spaces and a Theorem of Riesz	12
1.4. Gauges and Seminorms	19
1.5. Topology in Seminormed Linear Spaces	23
1.6. Problems	26
CHAPTER 2: TOPOLOGICAL LINEAR SPACES	31
2.0. Introduction	31
2.1. Topological Linear Spaces	31
2.2. Finite-Dimensional Topological Linear Spaces	35
2.3. Locally Convex Topological Linear Spaces	37
2.4. Seminorms and Convex Balanced Absorbing Sets	41
2.5. Frechet Spaces	46
2.6. Problems	49
CHAPTER 3: LINEAR TRANSFORMATIONS AND LINEAR FUNCTIONALS	55
3.0. Introduction	55
3.1. Linear Transformations	55
3.2. Some Basic Results Concerning Linear Transformations	60
3.3. Some Basic Results Concerning Linear Functionals	66
3.4. Problems	74

CHAPTER 4: THE HAHN-BANACH THEOREM: ANALYTIC FORM	80
4.0. Introduction	80
4.1. The Hahn-Banach Theorem: Analytic Form	80
4.2. Some Consequences of the Hahn-Banach Theorem	85
4.3. The Hahn-Banach Theorem and Abelian Semigroups of Transformations	90
4.4. Adjoint Transformations	96
4.5. Separability of V^*	98
4.6. Annihilators	99
4.7. Ideals in $L_1(\mathbb{R}, dt)$	102
4.8. Continuous Linear Functionals on $C([0,1])$	105
4.9. A Moment Problem	109
4.10. Helly's Theorem	111
4.11. Problems	116
CHAPTER 5: THE HAHN-BANACH THEOREM: GEOMETRIC FORM	126
5.0. Introduction	126
5.1. Linear Varieties and Hyperplanes	127
5.2. The Hahn-Banach Theorem: Geometric Form	132
5.3. Some Consequences of the Hahn-Banach Theorem Revisited	138
5.4. Some Further Geometric Consequences of the Hahn-Banach Theorem	140
5.5. Problems	142
CHAPTER 6: THE UNIFORM BOUNDEDNESS THEOREM	146
6.0. Introduction	146
6.1. The Baire Category Theorem and Osgood's Theorem	146
6.2. The Uniform Boundedness Theorem and the Banach-Steinhaus Theorem	148
6.3. The Strong Operator Topology	154
6.4. Local Membership in $L_q(\mathbb{R}, dt)$	158
6.5. A Result in the Theory of Summability	161

6.6.	Divergent Fourier Series	165
6.7.	Problems	169
CHAPTER 7: THE OPEN MAPPING AND CLOSED GRAPH THEOREMS		177
7.0.	Introduction	177
7.1.	Closed Mappings	177
7.2.	The Open Mapping Theorem	180
7.3.	The Closed Graph Theorem	187
7.4.	A Uniform Boundedness Theorem for Continuous Linear Functionals	191
7.5.	Some Results on Norms in $C([0,1])$.	192
7.6.	A Criterion for the Continuity of Linear Transformations on ℓ_2	194
7.7.	Separable Banach Spaces	196
7.8.	The Category of $L_1([-\pi,\pi], dt/2\pi)^{\wedge}$ in $C_0(\mathbb{Z})$	198
7.9.	Problems	199
CHAPTER 8: REFLEXIVITY		206
8.0.	Introduction	206
8.1.	Reflexive Spaces	206
8.2.	Uniform Convexity and Mil'man's Theorem	214
8.3.	Reflexivity of $L_p(X, S, \mu)$, $1 < p < \infty$	223
8.4.	Problems	229
CHAPTER 9: WEAK TOPOLOGIES		234
9.0.	Introduction	234
9.1.	F -topologies	235
9.2.	The Weak and Weak* Topologies	239
9.3.	Completeness in the Weak and Weak* Topologies	245
9.4.	The Banach-Alaoglu Theorem	254
9.5.	Banach Spaces as Spaces of Continuous Functions	263
9.6.	Banach Limits Revisited	264

9.7. Fourier Series of Functions in $L_p([-\pi, \pi], dt/2\pi), 1 < p < \infty$	267
9.8. Multipliers	272
9.9. Weak Compactness and Reflexivity	275
9.10. A Theorem Concerning the Adjoint Transformation	277
9.11. Problems	282
 CHAPTER 10: THE KREIN-ŠMULIAN AND EBERLEIN-ŠMULIAN THEOREMS	 290
10.0. Introduction	290
10.1. The Bounded Weak* Topology	290
10.2. The Krein-Šmulian Theorem	298
10.3. The Eberlein-Šmulian Theorem	303
10.4. Problems	312
 CHAPTER 11: THE KREIN-MIL'MAN THEOREM	 316
11.0. Introduction	316
11.1. Extreme Points and Extremal Sets	317
11.2. The Krein-Mil'man Theorem	322
11.3. $L_1(\mathbb{R}, dt)$ Is Not a Dual Space	326
11.4. The Stone-Weierstrass Theorem	327
11.5. Helson Sets	333
11.6. The Banach-Stone Theorem	337
11.7. Problems	345
 CHAPTER 12: FIXED POINT THEOREMS	 349
12.0. Introduction	349
12.1. The Fixed Point Property	349
12.2. Contraction Mappings	352
12.3. The Markov-Kakutani Fixed Point Theorem	354
12.4. The Picard Existence Theorem for Ordinary Differential Equations	357
12.5. Haar Measure on Compact Abelian Topological Groups	361
12.6. Problems	365

Contents	xi
CHAPTER 13: HILBERT SPACES	371
13.0. Introduction	371
13.1. Basic Definitions and Results	372
13.2. The Parallelogram and Polarization Identities	376
13.3. Some Other General Properties of Hilbert Spaces	382
13.4. The Orthogonal Decomposition Theorem and the Riesz Representation Theorem	384
13.5. Orthogonal Projections	392
13.6. Complete Orthonormal Sets	396
13.7. Fourier Analysis in $L_2([-\pi, \pi], dt/2\pi)$	409
13.8. Rademacher Functions	417
13.9. The Hilbert Space Adjoint	424
13.10. Self-adjoint and Unitary Transformations	429
13.11. The Mean Ergodic Theorem	434
13.12. A Theorem About H_2	441
13.13. Some Basic Results in Spectral Theory	446
13.14. Some Spectral Theory Results for Self-adjoint Transformations	452
13.15. A Spectral Decomposition Theorem for Compact Self-adjoint Transformations	458
13.16. Problems	470
REFERENCES	484
INDEX	489