Contents

51

52

58

59

		Contents
Preface		v
СНАРТЕ	R 1. Introduction to Inner Product Spaces	1
1.1	Some Prerequisite Material and Conventions	3
1.2	Inner Product Spaces	7
1.3	Linear Functionals, the Riesz Representation Theorem, and Adjoints	15
	Exercises 1	18
	References	19
СНАРТЕ	2. Orthogonal Projections and the Spectral Theorem fo	r 20
	Normai Transformations	20
2.1	The Complexification	21
2.2	Orthogonal Projections and Orthogonal Direct Sums	25
2.3	Unitary and Orthogonal Transformations	33
	Exercises 2	36
	References	37
СНАРТЕ	R 3. Normed Spaces and Metric Spaces	38
3.1	Norms and Normed Linear Spaces	39
3.2	Metrics and Metric Spaces	40
3.3	Topological Notions in Metric Spaces	43
3.4	Closed and Open Sets, Continuity, and Homeomorphisms	45
	Exercises 3	48
	Reference	49
	A Y A LOCAL DE LA MARIE CO	50
CHAPTEI	R 4. Isometries and Completion of a Metric Space	30

4.1 Isometries and Homeomorphisms

Exercises 4

Reference

4.2

Cauchy Sequences and Complete Metric Spaces

CHAPTER	5. Compactness in Metric Spaces	60
5.1	Nested Sequences and Complete Spaces	61
5.2	Relative Compactness &-Nets and Totally Bounded Sets	64
5.3	Countable Compactness and Sequential Compactness	67
	Exercises 5	72
	References	73
СНАРТЕР	6. Category and Separable Spaces	74
6.1	F_{σ} Sets and G_{δ} Sets	75
6.2	Nowhere-Dense Sets and Category	76
6.3	The Existence of Functions Continuous Everywhere, Differentiable Nowhere	80
6.4	Separable Spaces	82
	Exercises 6	84
	References	84
CHAPTE!	R 7. Topological Spaces	85
7.1	Definitions and Examples	86
7.2	Bases	90
7.3	Weak Topologies	93
7.4	Separation	96
7.5	Compactness	98
	Exercises 7	103
	References	107
СНАРТЕ	R 8. Banach Spaces, Equivalent Norms, and Factor Spaces	108
8.1	The Hölder and Minkowski Inequalities	109
8.2	Banach Spaces and Examples	112
8.3		118
8.4	Generated Subspaces and Closed Subspaces	121
8.5		122
8.6	-	126
8.7		129
8.8	-	131
	Exercises 8	133
	References	135

CHAPTER	9. Commutative Convergence, Hilbert Spaces, and Bessel's Inequality	136
9.1	Commutative Convergence	138
9.2	Norms and Inner Products on Cartesian Products of Normed	150
- · · -	and Inner Product Spaces	140
9.3	Hilbert Spaces	141
9.4	A Nonseparable Hilbert Space	143
9.5	Bessel's Inequality	144
9.6	Some Results from $L_2(0, 2\pi)$ and the Riesz-Fischer Theorem	146
9.7	Complete Orthonormal Sets	149
9.8	Complete Orthonormal Sets and Parseval's Identity	153
9.9	A Complete Orthonormal Set for $L_2(0, 2\pi)$	155
	Appendix 9	157
	Exercises 9	160
	References	161
CHAPTER	10. Complete Orthonormal Sets	162
10.1	Complete Orthonormal Sets and Parseval's Identity	163
10.2	The Cardinality of Complete Orthonormal Sets	166
10.3	A Note on the Structure of Hilbert Spaces	167
10.4	Closed Subspaces and the Projection Theorem for Hilbert	160
	Spaces Exercises 10	168
	References	172
	References	174
CHAPTER	11. The Hahn-Banach Theorem	175
11.1	The Hahn-Banach Theorem	176
11.2	Bounded Linear Functionals	182
11.3	The Conjugate Space	184
	Exercises 11	187
	Appendix 11. The Problem of Measure and the Hahn-Banach	
	Theorem	188
	Exercises 11 Appendix	195
•	References	195
CHAPTER	12. Consequences of the Hahn-Banach Theorem	196
12.1	Some Consequences of the Hahn-Banach Theorem	197
12.2	The Second Conjugate Space	203

12.3	The Conjugate Space of l_p	205
12.4	The Riesz Representation Theorem for Linear Functionals on	
	a Hilbert Space	209
12.5	Reflexivity of Hilbert Spaces	211
	Exercises 12	214
	References	215
CHAPTER	13. The Conjugate Space of $C[a, b]$	216
13.1	A Representation Theorem for Bounded Linear Functionals on $C[a, b]$	218
13.2	A List of Some Spaces and Their Conjugate Spaces	227
	Exercises 13	227
	References	229
	14 Week Communication and Downsted Linear	
CHAPTER	14. Weak Convergence and Bounded Linear Transformations	230
14.1	Weak Convergence	231
14.2	Bounded Linear Transformations	238
	Exercises 14	243
	References	244
CHAPTER	15. Convergence in $L(X, Y)$ and the Principle of Uniform Boundedness	245
15.1		246
15.2	•	250
15.3	•	253
	Exercises 15	257
	References	258
CHAPTER	16. Closed Transformations and the Closed Graph Theorem	259
16.1	The Graph of a Mapping	260
16.2		261
160	Theorem Some Consequences of the Bounded Inverse Theorem	271
16.3	Appendix 16. Supplement to Theorem 16.5	273
	Exercises 16	274
	References	275

CHAPTER	17. Closures, Conjugate Transformations, and Complete Continuity	276
17.1	The Closure of a Linear Transformation	277
17.2	A Class of Linear Transformations that Admit a Closure	279
17.3	The Conjugate Map of a Bounded Linear Transformation	281
17.4	Annihilators	283
17.5	Completely Continuous Operators; Finite-Dimensional Operators	286
17.6	Further Properties of Completely Continuous Transformations	289
	Exercises 17	294
	References	295
CHAPTER	18. Spectral Notions	296
18.1	Spectra and the Resolvent Set	297
18.2	The Spectra of Two Particular Transformations	300
18.3	Approximate Proper Values	304
	Exercises 18	304
	References	306
CHAPTER	19. Introduction to Banach Algebras	307
19.1	Analytic Vector-Valued Functions	308
19.2	Normed and Banach Algebras	311
19.3	Banach Algebras with Identity	315
19.4	An Analytic Function – the Resolvent Operator	319
19.5	Spectral Radius and the Spectral Mapping Theorem for Polynomials	322
19.6	The Gelfand Theory	327
19.7	Weak Topologies and the Gelfand Topology	336
19.8	Topological Vector Spaces and Operator Topologies	342
	Exercises 19	348
	References	350
CHAPTER	20. Adjoints and Sesquilinear Functionals	351
20.1	The Adjoint Operator	352
20.2	Adjoints and Closures	355
20.3	Adjoints of Bounded Linear Transformations in Hilbert Spaces	362

	~ — h .	
20.4	Sesquilinear Functionals	367
	Exercises 20	373
	References	374
CHAPTER	21. Some Spectral Results for Normal and Completely Continuous Operators	375
21.1	A New Expression for the Norm of $A \in L(X, X)$	376
21.2	Normal Transformations	379
21.3	Some Spectral Results for Completely Continuous Operators	383
21.4	Numerical Range	386
	Exercises 21	389
	Appendix to Chapter 21. The Fredholm Alternative Theorem and the Spectrum of a Completely Continuous Transformation	390
	A.1 Motivation	391
	A.2 The Fredholm Alternative Theorem	393
	References	405
CHAPTER	22. Orthogonal Projections and Positive Definite	
	Operators	406
22.1	Properties of Orthogonal Projections	407
22.2	Products of Projections	409
22.3	Positive Operators	411
22.4	Sums and Differences of Orthogonal Projections	412
22.5	The Product of Positive Operators	415
	Exercises 22	417
	References	418
CHAPTER	23. Square Roots and a Spectral Decomposition Theorem	419
23.1	Square Root of Positive Operators	420
23.2	Spectral Theorem for Bounded, Normal, Finite-Dimensional	
	Operators	426
	Exercises 23	430
	References	431
CHAPTER	24. Spectral Theorem for Completely Continuous Normal Operators	432
	-, A	
24.1	Infinite Orthogonal Direct Sums: Infinite Series of Transformations	433

\sim	ファ	JTI	FN	тς

>	۲i	i	i
-	_	-	-

	CONTENTS	XII
24.2	Spectral Decomposition Theorem for Completely Continuous Normal Operators	438
	Exercises 24	442
	References	443
CHAPTER	25. Spectral Theorem for Bounded, Self-Adjoint Operators	444
25.1	A Special Case – the Self-Adjoint, Completely Continuous Operator	445
25.2	Further Properties of the Spectrum of Bounded, Self-Adjoint Transformations	448
25.3	Spectral Theorem for Bounded, Self-Adjoint Operators	450
	Exercises 25	458
	References	459
CHAPTER	26. A Second Approach to the Spectral Theorem for Bounded, Self-Adjoint Operators	460
26.1	A Second Approach to the Spectral Theorem for Bounded,	
	Self-Adjoint Operators	461
	Exercises 26	469
	References	470
CHAPTER	27. A Third Approach to the Spectral Theorem for Bounded, Self-Adjoint Operators and Some	
	Consequences	471
27.1	A Third Approach to the Spectral Theorem for Bounded, Self-Adjoint Operators	472
27.2	Two Consequences of the Spectral Theorem	478
21.2	Exercises 27	483
	References	483
CHAPTER	28. Spectral Theorem for Bounded, Normal Operators	484
28.1	The Spectral Theorem for Bounded, Normal Operators on	
	a Hilbert Space	485
28.2	Spectral Measures; Unitary Transformations	488
	Exercises 28	493
	References	493

CONTENTS

CHAPTER	29. Spectral Theorem for Unbounded, Self-Adjoint Operators	494
29.1	Permutativity	495
29.2	The Spectral Theorem for Unbounded, Self-Adjoint Operators	498
29.3	A Proof of the Spectral Theorem Using the Cayley Transform	516
29.4	A Note on the Spectral Theorem for Unbounded Normal	
	Operators	520
	Exercises 29	521
	References	522
D:11:	<i>1</i>	523
Bibliograp	ny	
Index of S	ymbols	525
Subject Inc	dex	527