Contents

Introdu	action	1
1.	The Subject Matter	1
2.	The Book. Conventions, Prerequisites, Etc.	7
3.	Acknowledgements	9
Chapte	er 1. Retractions, Extensions and Selections	11
1.	Absolute Lipschitz Retracts	11
2.	Extension of Maps on Hilbert Space	18
3.	Michael's Selection Theorem	21
4.	Lipschitz and Uniform Selections	26
5.	Notes and Remarks	31
Chapte	er 2. Retractions, Extensions and Selections (Special Topics)	35
1.	Approximation of Uniformly Continuous Functions	35
2.	The Nearest Point Map	40
3.	The Contraction Extension Property	44
4.	The Steiner Point	48
5.	Simultaneously Continuous Maps	53
6.	Notes and Remarks	58
Chapte	er 3. Fixed Points	61
1.		61
2.	Lipschitz Maps	63
3.	Nonexpansive Maps	65
4.	Approximation of Fixed Points	70
5.	Notes and Remarks	79
Chapte	Chapter 4. Differentiation of Convex Functions	
1.	Basic Definitions and Results	83
2.	Convex Functions	85
3.	Notes and Remarks	96
Chapte	er 5. The Radon-Nikodým Property	99
1.	Vector Measures and Integration of Vector-Valued Functions	99
2.	The Radon-Nikodým Property	102
3.	Differentiability, Trees and the RNP	110
4.	Examples Related to the RNP	114
5.	Notes and Remarks	121
Chapte	er 6. Negligible Sets and Gâteaux Differentiability	125
1.		125
2.	Gaussian Measures	135

x CONTENTS

3. Gauss Null Sets	141
4. Gâteaux Differentiability of Lipschitz Functions	153
5. Examples Related to Fréchet Differentiability	156
6. Notes and Remarks	161
7. Summary	166
Chapter 7. Lipschitz Classification of Banach Spaces	169
1. Linearization of Lipschitz Maps	169
2. Applications and Examples	174
3. Notes and Remarks	183
Chapter 8. Uniform Embeddings into Hilbert Space	185
1. Positive Definite and Negative Definite Functions	185
2. Uniform Embeddings into Hilbert Space	190
3. Notes and Remarks	195
Chapter 9. Uniform Classification of Spheres	197
1. The Mazur Map	197
2. Unit Spheres of Banach Lattices	199
3. Applications of the Complex Interpolation Method	204
4. Spheres and Balls	$\frac{206}{212}$
5. Stable Metrics	215
6. Notes and Remarks	210
Chapter 10. Uniform Classification of Banach Spaces	219
1. Reduction to Lipschitz and Linear Maps	219
2. Approximate Midpoints	229
3. Discrete Nets	236
4. Nonisomorphic Uniformly Homeomorphic Spaces	244
5. Uniform Types That Determine a Finite Number of Linear Structures	240
6. Notes and Remarks	253
Chapter 11. Nonlinear Quotient Maps	261
1. Surjective Lipschitz and Smooth Maps	261
2. Nonlinear Quotient Maps	268
3. Notes and Remarks	277
Chapter 12. Oscillation of Uniformly Continuous Functions on Unit	
Spheres of Finite-Dimensional Subspaces	283
1. Dvoretzky's Theorem	28
2. Krivine's Theorem	289
3. Notes and Remarks	298
Chapter 13. Oscillation of Uniformly Continuous Functions on Unit	
Spheres of Infinite-Dimensional Subspaces	301
1. Preliminary Results	30:
2. Existence of Subspaces Isomorphic to l_p or c_0	307
3. Uniformly Continuous Functions on the Unit Sphere of c_0	312
4. Asymptotic Biorthogonal Systems	320

CONTENTS	xi
----------	----

481

5. Asymptotic Biorthogonal Systems in l_p 6. Notes and Remarks	328 333	
 Chapter 14. Perturbations of Local Isometries Isometries Quasi-Isometries, Injectivity Approximation of Quasi-Isometries by Isometries Approximation of the Derivative Notes and Remarks 	341 341 343 348 352 356	
Chapter 15. Perturbations of Global Isometries 1. The Hyers-Ulam Problem 2. Larger Perturbations 3. Notes and Remarks	359 359 363 370	
Chapter 16. Twisted Sums 1. Quasi-Linear Functions 2. Twisted Sums of Hilbert Spaces 3. Notes and Remarks	373 373 380 389	
Chapter 17. Group Structure on Banach Spaces 1. Banach Groups 2. Hilbert's Fifth Problem in Infinite Dimensions 3. Notes and Remarks	391 391 399 406	
Appendices A. Convexity B. Partitions of Unity C. Invariant Means D. Measure and Probability E. Bases and Lattices F. Local Structure of Infinite-Dimensional Spaces G. Quantitative Theory of Banach Spaces H. Quasi-Normed Spaces I. The Complex Interpolation Method J. Operators on Hilbert Space	409 409 416 417 418 429 436 440 445 448	
Bibliography		

 ${\bf Index}$