

## TABLE OF CONTENTS

INTRODUCTION	v
CHAPTER 0. NOTATIONS AND PRELIMINARIES	1
<b>PART 1. FUNCTIONAL ANALYSIS</b>	
CHAPTER I. BAIRE'S PROPERTY AND ITS CONSEQUENCES	7
CHAPTER II. INFINITE-DIMENSIONAL NORMED SPACES	17
§ 1. Hahn-Banach Theorem : Analytic form	19
§ 2. Weak topology on the dual $E^*$	23
§ 3. The bidual $E^{**}$ and the weak topology on $E$	25
§ 4. The separation of convex subsets : the geometric form of Hahn-Banach Theorem	29
§ 5. Closed convex sets and bounded sets in the weak and in the strong topologies	34
§ 6. Weak and strong continuity - Transposition	37
§ 7. Duality between subspaces and quotients for weak and strong topologies	41
CHAPTER III. REFLEXIVE BANACH SPACES ; SEPARABLE BANACH SPACES	49
§ 1. Reflexivity	49
§ 2. Separability	55
§ 3. The utilization of convergent sequences in order to define a topology. Weak compactness	59
<b>PART 2. THE STRUCTURE OF SOME COMMON BANACH SPACES</b>	
CHAPTER I. HILBERT SPACES	69

CHAPTER II. SCHAUDER BASES IN BANACH SPACES	79
§ 1. Schauder bases	79
§ 2. Unconditional Schauder bases	88
CHAPTER III. COMPLEMENTED SUBSPACES IN BANACH SPACES	99
CHAPTER IV. THE BANACH SPACES $\ell_p (1 \leq p \leq +\infty)$ AND $c_0$	107
§ 1. Subspaces of $\ell_p (1 \leq p \leq +\infty)$ and $c_0$	107
§ 2. Some particular properties of $\ell_1$ and $\ell_\infty$	114
CHAPTER V. EXTREME POINTS OF COMPACT CONVEX SETS AND THE BANACH SPACES $\mathcal{C}(K)$	123
§ 1. Extreme points of compact convex sets	123
§ 2. The Banach spaces $\mathcal{C}(K)$	126
CHAPTER VI. THE BANACH SPACES $L_p(\Omega, \mathcal{A}, \mu)$ ( $1 \leq p < +\infty$ )	137
§ 1. Subspaces of $L_p (1 \leq p < +\infty)$	140
§ 2. The space $L_1$	153
§ 3. Banach valued functions	167
<b>PART 3. SOME METRIC PROPERTIES IN BANACH SPACES</b>	
CHAPTER I. STRICT CONVEXITY AND SMOOTHNESS	175
§ 1. Strict convexity	175
§ 2. Smoothness and Gâteaux-differentiability of the norm	177
§ 3. Duality between strict convexity and smoothness	184
CHAPTER II. UNIFORM CONVEXITY AND UNIFORM SMOOTHNESS	189
§ 1. Uniform convexity	189
§ 2. Uniform smoothness and Uniform Fréchet Differentiability	204
<b>PART 4. THE GEOMETRY OF SUPER-REFLEXIVE BANACH SPACES</b>	
CHAPTER I. FINITE REPRESENTABILITY AND SUPER-PROPERTIES OF BANACH SPACES	217
§ 1. Finite representability and Ultrapowers of Banach spaces	217
§ 2. Super-properties of Banach spaces ; super-reflexive spaces	225
§ 3. The Tree Properties	226

CHAPTER II. BASIC SEQUENCES IN SUPER-REFLEXIVE BANACH SPACES	243
CHAPTER III. UNIFORMLY NON-SQUARE AND J-CONVEX BANACH SPACES	255
§ 1. Uniformly non-square Banach Spaces	255
§ 2. J-convex Banach Spaces	261
CHAPTER IV. RENORMING SUPER-REFLEXIVE BANACH SPACES	273
§ 1. Banach-valued Martingales	273
§ 2. A sequence of norms on a super-reflexive space	277
BIBLIOGRAPHY	301
INDEX	305