

Contents

Preface	vii
Prologue	xix
Contents of AMS Volume 109	xxvii
1 Banach Spaces and Fixed-Point Theorems	1
1.1 Linear Spaces and Dimension	2
1.2 Normed Spaces and Convergence	7
1.3 Banach Spaces and the Cauchy Convergence Criterion	10
1.4 Open and Closed Sets	15
1.5 Operators	16
1.6 The Banach Fixed-Point Theorem and the Iteration Method	18
1.7 Applications to Integral Equations	22
1.8 Applications to Ordinary Differential Equations	24
1.9 Continuity	26
1.10 Convexity	29
1.11 Compactness	33
1.12 Finite-Dimensional Banach Spaces and Equivalent Norms	42
1.13 The Minkowski Functional and Homeomorphisms	45
1.14 The Brouwer Fixed-Point Theorem	53
1.15 The Schauder Fixed-Point Theorem	61
1.16 Applications to Integral Equations	62
1.17 Applications to Ordinary Differential Equations	63

1.18	The Leray–Schauder Principle and a priori Estimates	64
1.19	Sub- and Supersolutions, and the Iteration Method in Ordered Banach Spaces	66
1.20	Linear Operators	70
1.21	The Dual Space	74
1.22	Infinite Series in Normed Spaces	76
1.23	Banach Algebras and Operator Functions	76
1.24	Applications to Linear Differential Equations in Banach Spaces	80
1.25	Applications to the Spectrum	82
1.26	Density and Approximation	84
1.27	Summary of Important Notions	88
2	Hilbert Spaces, Orthogonality, and the Dirichlet Principle	101
2.1	Hilbert Spaces	105
2.2	Standard Examples	109
2.3	Bilinear Forms	120
2.4	The Main Theorem on Quadratic Variational Problems	121
2.5	The Functional Analytic Justification of the Dirichlet Principle	125
2.6	The Convergence of the Ritz Method for Quadratic Variational Problems	140
2.7	Applications to Boundary-Value Problems, the Method of Finite Elements, and Elasticity	145
2.8	Generalized Functions and Linear Functionals	156
2.9	Orthogonal Projection	165
2.10	Linear Functionals and the Riesz Theorem	167
2.11	The Duality Map	169
2.12	Duality for Quadratic Variational Problems	169
2.13	The Linear Orthogonality Principle	172
2.14	Nonlinear Monotone Operators	173
2.15	Applications to the Nonlinear Lax–Milgram Theorem and the Nonlinear Orthogonality Principle	174
3	Hilbert Spaces and Generalized Fourier Series	195
3.1	Orthonormal Series	199
3.2	Applications to Classical Fourier Series	203
3.3	The Schmidt Orthogonalization Method	207
3.4	Applications to Polynomials	208
3.5	Unitary Operators	212
3.6	The Extension Principle	213
3.7	Applications to the Fourier Transformation	214
3.8	The Fourier Transform of Tempered Generalized Functions	219

4 Eigenvalue Problems for Linear Compact Symmetric Operators	229
4.1 Symmetric Operators	230
4.2 The Hilbert–Schmidt Theory	232
4.3 The Fredholm Alternative	237
4.4 Applications to Integral Equations	240
4.5 Applications to Boundary-Eigenvalue Value Problems	245
5 Self-Adjoint Operators, the Friedrichs Extension and the Partial Differential Equations of Mathematical Physics	253
5.1 Extensions and Embeddings	260
5.2 Self-Adjoint Operators	263
5.3 The Energetic Space	273
5.4 The Energetic Extension	279
5.5 The Friedrichs Extension of Symmetric Operators	280
5.6 Applications to Boundary-Eigenvalue Problems for the Laplace Equation	285
5.7 The Poincaré Inequality and Rellich’s Compactness Theorem	287
5.8 Functions of Self-Adjoint Operators	293
5.9 Semigroups, One-Parameter Groups, and Their Physical Relevance	298
5.10 Applications to the Heat Equation	305
5.11 Applications to the Wave Equation	309
5.12 Applications to the Vibrating String and the Fourier Method	315
5.13 Applications to the Schrödinger Equation	323
5.14 Applications to Quantum Mechanics	327
5.15 Generalized Eigenfunctions	343
5.16 Trace Class Operators	347
5.17 Applications to Quantum Statistics	348
5.18 C^* -Algebras and the Algebraic Approach to Quantum Statistics	357
5.19 The Fock Space in Quantum Field Theory and the Pauli Principle	363
5.20 A Look at Scattering Theory	368
5.21 The Language of Physicists in Quantum Physics and the Justification of the Dirac Calculus	373
5.22 The Euclidean Strategy in Quantum Physics	379
5.23 Applications to Feynman’s Path Integral	385
5.24 The Importance of the Propagator in Quantum Physics	394
5.25 A Look at Solitons and Inverse Scattering Theory	406
Epilogue	425
Appendix	429

References	443
Hints for Further Reading	455
List of Symbols	459
List of Theorems	465
List of the Most Important Definitions	467
Subject Index	471