Contents

Preface

1 Linear Operators in Banach Spaces **1**
1.1 Metric Spaces 1
1.2 Vector Spaces 6
1.3 Banach Spaces 8
1.4 Linear Operators 11
1.5 Duals 16
1.6 Spectrum 23
1.7 Compact Linear Operators 29
1.8 Boundary Value Problems for Linear ODEs 39
1.9 Exercises 43

2 Linear Operators in Hilbert Spaces **47**
2.1 Orthonormal Sets 47
2.2 Adjoints 54
2.3 Accretive Operators 58
2.4 Weak Solutions 62
2.5 Example: Constant Coefficient PDEs 68
2.6 Self-adjoint Operators 70
2.7 Example: Sturm-Liouville Problem 76
2.8 Sectorial Forms 80
2.9 Example: Harmonic Oscillator and Hermite Functions 88
2.10 Example: Completeness of Bessel Functions 92
2.11 Example: Finite Element Method 96
2.12 Friedrichs Extension 99
2.13 Exercises 104

3 Sobolev Spaces **109**
3.1 Introduction 109
3.2 Fourier Transform 114
3.3 Distributions 122

3.4 Weak Derivatives 126
3.5 Definition and Basic Properties of Sobolev Spaces 135
3.6 Imbeddings of $W^{m,p}(\Omega)$ 140
3.7 Elliptic Problems 153
3.8 Regularity of Weak Solutions 160
3.9 Exercises 163

4 Semigroups of Linear Operators **167**
4.1 Introduction 167
4.2 Bochner Integral 171
4.3 Basic Properties of Semigroups 176
4.4 Example: Wave Equation 187
4.5 Sectorial Operators and Analytic Semigroups 192
4.6 Invariant Subspaces 204
4.7 The Inhomogeneous Problem - Part I 209
4.8 Exercises 212

5 Weakly Nonlinear Evolution Equations **215**
5.1 Introduction 215
5.2 Basic Theory 218
5.3 Example: Nonlinear Heat Equation 222
5.4 Approximation for Evolution Equations 224
5.5 Example: Finite Difference Method 233
5.6 Example: Galerkin Method for Parabolic Equations 236
5.7 Example: Galerkin Method for the Wave Equation 239
5.8 Friedrichs Extension and Galerkin Approximations 243
5.9 Exercises 245

6 Semilinear Parabolic Equations **247**
6.1 Fractional Powers of Operators 247
6.2 The Inhomogeneous Problem - Part II 254
6.3 Global Version 257
6.4 Main Results 259
6.5 Example: Navier-Stokes Equations 267
6.6 Example: A Stability Problem 271
6.7 Example: A Classical Solution 273
6.8 Dynamical Systems 276
6.9 Example: The Chafee-Infante Problem 279
6.10 Exercises 282

Bibliography **285**

List of Symbols **289**

Index **291**