

CONTENTS

<i>FOREWORD TO THE FIRST ENGLISH EDITION</i>	10
<i>FOREWORD TO THE SECOND ENGLISH EDITION</i>	11
<i>SUPPLEMENTS TO VOLUME I</i>	12
(A) Supplement to Part I, Chapter VI	12
(B) Supplement to Part III, Chapter VIII	13
 PART IV — AN OUTLINE OF THE GENERAL THEORY OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS	
CHAPTER I. Homogeneous equations	
§ 1. Introductory remarks	16
§ 2. Characteristic equations	16
§ 3. On exponential functions	17
§ 4. Logarithms	18
§ 5. Multiple roots of the characteristic equation	19
§ 6. The general solution	20
§ 7. Theorem on uniqueness of solution	22
§ 8. The logarithmic equation	24
§ 9. Linear differential expressions	25
§ 10. Operations on linear differential expressions	26
§ 11. Characteristic polynomials of linear differential expressions	27
§ 12. Pure equations	27
§ 13. Mixed equations	27
§ 14. Adapting the solution to given initial, boundary and other conditions	28
 CHAPTER II. Non-homogeneous equations	
§ 15. The general solution of a non-homogeneous equation	31
§ 16. The case where the right side is a polynomial	32
§ 17. The case where the right side is an exponential function	33
§ 18. The case where the right side is a product of a polynomial and an exponential function	34
§ 19. The case where the right side is a linear combination of two functions	35
§ 20. The case where the right side is a trigonometric function	35
§ 21. Adapting the solution to additional conditions	36
 CHAPTER III. Applications to partial differential equations	
§ 22. Reducing partial operational equations to operational equations	39
§ 23. Remarks on additional conditions	44
§ 24. An incorrect solution	45
§ 25. Explaining the apparent contradiction	46
§ 26. The Cauchy conditions and the question of their being equivalent to the general conditions	48
§ 27. Solving restrictive equations	50

§ 28. The question of the equivalence of a partial equation and an operational equation	52
§ 29. Further examples of solving partial equations	53
§ 30. General remarks on solving partial equations by the operational method	56
§ 31. Mixed problems	59

PART V—INTEGRAL OPERATIONAL CALCULUS

CHAPTER I. The integral of an operational function and its applications

§ 1. Operational functions of class (\mathcal{K})	66
§ 2. The definition of the integral	67
§ 3. Properties of the integral	68
§ 4. Operational functions of two variables	70
§ 5. Cutting down a function	72
§ 6. The integral form of a certain particular solution of the logarithmic differential equation	74
§ 7. Application to the equation of a vibrating string	76
§ 8. Application of infinite series and definite integrals	79

CHAPTER II. Integral transformations

§ 9. The Laplace transform	82
§ 10. The Laplace transform as a basis for the operational calculus	83
§ 11. A comparison of the direct method and the method of Laplace transform	84
§ 12. Other related methods	85

PART VI—ADVANCED TOPICS IN THE OPERATIONAL CALCULUS

CHAPTER I. Definition of operators in terms of abstract algebra

§ 1. Commutative ring	86
§ 2. Quotient field	87
§ 3. Operators	88
§ 4. Rings with divisors of zero	89
§ 5. Periodic operators	89
§ 6. The Fourier series of a periodic operator	90

CHAPTER II. Locally integrable functions

§ 1. The convolution of integrable functions	92
§ 2. Properties of this convolution	92
§ 3. Locally integrable functions as operators	93
§ 4. Functions of class \mathcal{K}	93
§ 5. Absolutely continuous functions	95
§ 6. The ring of locally integrable functions	95

CHAPTER III. Operators as generalized functions

§ 1. Introduction	97
§ 2. The class of left-bounded functions on the whole real line	97
§ 3. The support number of an operator	100
§ 4. The support of continuous functions	101
§ 5. Approximate identities	102
§ 6. Convolution of functions defined on an arbitrary open set	103
§ 7. Almost uniform convergence on open sets	104
§ 8. Regular operators	105
§ 9. Examples of regular operators	106

§ 10. An example of an operator which is not regular	107
§ 11. The restriction of operators to an open set	108
§ 12. A characterization of support	110
§ 13. Derivatives of an operator	111
§ 14. The proof of Theorem A	111
§ 15. The proof of Theorem B	112
§ 16. Some examples of derivatives	113
§ 17. Regular convergence of operators	116
§ 18. A comparison with other types of generalized functions	118
§ 19. Defining sequences	119
§ 20. Generalized functions on the whole real line	120

CHAPTER IV. Convergence in the space of operators

§ 1. Introduction	122
§ 2. Approximation by convolution	122
§ 3. Infinite convolution products	123
§ 4. Continuous convolution multiples	125
§ 5. Common denominators for a sequence of operators	126
§ 6. The diagonal subsequence property	127
§ 7. The space of operators which have positive support numbers	127
§ 8. A convergent sequence	129
§ 9. Topological convergence	130
§ 10. The connection between sequential and topological convergence	131
§ 11. A topological convergence in the space of operators	134
§ 12. Functionals on the space \mathcal{F}_0	135
§ 13. A characterization of operator convergence type I'	137
§ 14. Metrizable topological vector spaces	138
§ 15. The space \mathcal{F}_0 as a metric space	141
§ 16. The space of operators as the union of metric spaces	142
§ 17. Bounded sets in the space of operators	143
§ 18. Precompact sets in the space of operators	144
§ 19. Precompact collections in \mathcal{C}	144
§ 20. Convolution takes bounded sets into precompact sets	145
§ 21. Any bounded set of operators is precompact	146
§ 22. Periodic operators	149
§ 23. Continuous operator functions	150
§ 24. Differentiable operator functions	152
§ 25. Integrals of operator functions	155

CHAPTER V. Power series of operators

§ 1. Power series of integrable functions	156
§ 2. A more general case	157
§ 3. Some particular power series	159
§ 4. The classes $C\{M_n\}$	163
§ 5. The logarithmically convex hull of a sequence	165
§ 6. Power series in the operator s	168
§ 7. The support of power series in s	168

CHAPTER VI. Laplace transform

§ 1. Fundamental properties of the Laplace transform	172
§ 2. Complex inversion formula	173

§ 3. Post's inversion formula	175
§ 4. Laplace transform of convolution	177
§ 5. Finite Laplace transform	177
§ 6. The Laplace transform of a function which vanishes on an initial interval	178
§ 7. The theorem of Titchmarsh for $f = g$	179
§ 8. The full theorem of Titchmarsh	180
§ 9. Convex sets in R^k	181
§ 10. Functions with compact support in R^k	183
§ 11. A theorem of Lions	183
§ 12. The proof of the theorem of Lions	185
§ 13. A Titchmarsh type theorem for R^k	186
§ 14. Operators in several variables	189

CHAPTER VII. A class of Dirichlet series

§ 1. Introduction	190
§ 2. Hirschman-Widder functions	191
§ 3. A sequence of translated Hirschman-Widder functions	191
§ 4. Entire function generated by a given sequence of exponents	192
§ 5. Generalized Euler's gamma function	195
§ 6. Generalized Phragmén's discontinuity factor	197
§ 7. A theorem on bounded moments	198
§ 8. Titchmarsh's theorem on convolution	200

CHAPTER VIII. The exponential function $\exp(-\lambda s^\alpha)$

§ 1. Introduction	202
§ 2. Case $\alpha < 0$	202
§ 3. Case $\alpha = 0$	203
§ 4. Case $0 < \alpha < 1$	203
§ 5. Expansion into Taylor's series	207
§ 6. Function $U_\alpha(t)$	208
§ 7. Real formula of $U_\alpha(t)$	209
§ 8. Properties of $U_\alpha(t)$ on the real axis	211
§ 9. The case of $\alpha > 1$ and real λ	212
§ 10. Real and imaginary part of an operator	214
§ 11. The case of $\alpha > 1$ and a complex, but not imaginary λ	215
§ 12. The case of $\alpha \geq 1$ and imaginary λ	215
§ 13. The case of $\alpha = 1$	217
§ 14. Table of existence	217

PART VII—FORMULAE AND TABLES

I. Special functions

1. Gamma function of Euler	218
2. Error function	218
3. Bessel functions	218

II. Formulae of the operational calculus 219

III. Electrotechnical applications

1. Equation of the circuit	223
2. Stationary current	223
3. Table of simple quadripoles and their matrices	224

IV. Tables of functions

1. Gamma function of Euler $\Gamma(\lambda)$	225
2. Error function $\operatorname{erf} \lambda$	225
3. Bessel function $J_0(\lambda)$	226
4. Bessel function $J_1(\lambda)$	226
5. Functions $J_0(i\lambda)$ and $-iJ_1(i\lambda)$	227

APPENDIX—SOME TOPICS FROM REAL ANALYSIS

§ 1. Introduction	228
§ 2. Some properties of R^k	228
§ 3. A theorem of Carathéodory	231
§ 4. The Riemann–Lebesgue lemma	235
§ 5. Metric and normed spaces	237
§ 6. The Hahn–Banach theorem	238
§ 7. A corollary to the Hahn–Banach theorem	241
§ 8. The Birkhoff–Kakutani theorem for topological vector spaces	242
§ 9. The Arzelà–Ascoli theorem	244
§ 10. Two lemmas	247
§ 11. The proof of the Denjoy–Carleman theorem	248

<i>Answers to problems</i>	252
<i>Bibliography</i>	254
<i>Index</i>	258
<i>Other titles in the series</i>	261