Table of Contents

Preface ix

Notations xiii

1 G-convergence of Abstract Operators 1

1.1 Preliminaries . . . 1

1.1.1 Multivalued Monotone Operators . . . 1

1.1.2 Single-Valued Operators of Monotone Type . . . 5

1.1.3 Convergence in the Sense of Kuratowski . . . 8

1.2 G-convergence of Monotone Operators . . . 10

1.2.1 Classes of Operators . . . 10

1.2.2 G-convergence and G-compactness . . . 14

1.2.3 Comparision of Different Types of Operator Convergence . . . 18

1.2.4 Some Special Propeties of G-convergence . . . 24

1.3 G-convergence of Abstract Parabolic Operators . . . 25

1.3.1 Abstract Parabolic Operators . . . 25

1.3.2 G-compactness . . . 33

1.3.3 Properties of G-convergence . . . 36

1.3.4 Time Homogenization of Abstract Parabolic Operators . . . 41

Comments . . . 44

2 Strong G-convergence of Nonlinear Elliptic Operators 45

2.1 Nonlinear Elliptic Operators . . . 45

2.1.1 Measurable Multivalued Functions . . . 45

2.1.2 Multivalued Monotone Elliptic Operators . . . 49

2.1.3 Some Classes of Single-Valued Elliptic Operators . . . 58

2.2 Strong G-convergence for Multivalued Elliptic Operators . . . 62

2.2.1 Definition of Strong G-convergence . . . 62

2.2.2 Strong G-compactness . . . 65

2.2.3 Additional Results . . . 68

2.2.4 Variational Problems . . . 76

2.2.5 Other Boundary Conditions . . . 80

2.3 Strong G-convergence for Single-Valued Elliptic Operators . . . 83

2.3.1 Main Results . . . 83
2.3.2 Proofs of Main Results: Particular Case . . . 88
2.3.3 Proofs of Main Results: General Case . . . 98
2.4 Further Results on Strong G-convergence . . . 108
2.4.1 Criteria for Strong G-convergenvce . . . 108
2.4.2 Stability and Comparison Results . . . 112
2.4.3 One-Dimentional Case . . . 118
2.5 Strong Nonlinearity in Lower Order Term . . . 120
Comments . . . 129

3 Homogenization of Elliptic Operators **131**
3.1 Random Homogeneous Fields . . . 131
3.1.1 Definitions and Main Properties . . . 131
3.1.2 Vector Fields and Compensated Compactness . . . 136
3.1.3 Random Vector Fields . . . 137
3.2 Homogenization of Random Elliptic Operators . . . 141
3.2.1 Multivalued Monotone Operators and Auxiliary Problem . . . 141
3.2.2 Homogenization Theorem . . . 146
3.2.3 Properties of Homogenizated Operators . . . 149
3.2.4 Single-Valued Elliptic Operators . . . 152
3.3 Almost Periodic Homogenization . . . 155
3.3.1 Almost Periodic Functions . . . 155
3.3.2 Individual Homogenization . . . 159
3.4 One-Dimentional Problems . . . 163
3.5 Additional Results . . . 166
3.5.1 Operators with Strong Nonlinearity . . . 166
3.5.2 Correctors . . . 170
Comments . . . 172

4 Nonlinear Parabolic Operators **173**
4.1 Strong G-convergence . . . 173
4.1.1 Main Definitions . . . 173
4.1.2 Monotone Operators . . . 177
4.1.3 General Parabolic Operators . . . 183
4.1.4 Further Results . . . 187
4.2 Homogenization . . . 189
4.2.1 Setting of the Problem . . . 189
4.2.2 Self-Similar Case . . . 192
4.2.3 Non Self-Similar Cases . . . 195
4.2.4 Spatial Homogenization . . . 199
4.2.5 Time Homogenization . . . 200
4.3 An Equation of Nonstationary Filtration . . . 203

Comments 212

A Homogenization of Nonlinear Difference Schemes **213**
A.1 Mesh Functions 214
A.2 G-convergence 216
A.3 Homogenization 220

B Open Problems **224**

References **229**

Index **248**