

Contents

Preface	9
Introduction	11
1 The operator of singular integration	15
1.1 Notations, definitions and auxiliary statements	15
1.1.1 The operator of singular integration	15
1.1.2 The space $L_p(\Gamma, \rho)$	17
1.1.3 Interpolation theorems	18
1.2 The boundedness of the operator S_Γ in the space $L_p(\Gamma)$ with Γ being a simple curve	18
1.3 Nonsimple curves	24
1.4 Integral operators in weighted L_p spaces	30
1.5 Unbounded curves	37
1.6 The operator of singular integration in spaces of Hölder continuous functions	39
1.7 The operator S_Γ^*	41
1.8 Exercises	46
Comments and references	49
2 One-sided invertible operators	51
2.1 Direct sum of subspaces	51
2.2 The direct complement	54
2.3 Linear operators. Notations and simplest classes	56
2.4 Projectors connected with the operator of singular integration	57
2.5 One-sided invertible operators	63
2.6 Singular integral operators and related operators	66
2.7 Examples of one-sided invertible singular integral operators	69
2.8 Two lemmas on the spectrum of an element in a subalgebra of a Banach algebra	73
2.9 Subalgebras of a Banach algebra generated by one element	76
2.10 Exercises	78

Comments and references	80
3 Singular integral operators with continuous coefficients	81
3.1 The index of a continuous function	81
3.2 Singular integral operators with rational coefficients	83
3.3 Factorization of functions	88
3.4 The canonical factorization in a commutative Banach algebra	91
3.5 Proof of the factorization theorem	96
3.6 The local factorization principle	99
3.7 Operators with continuous coefficients	102
3.8 Approximate solutions of singular integral equations	105
3.9 Generalized factorizations of continuous functions	108
3.10 Operators with continuous coefficients (continuation)	111
3.11 Additional facts and generalizations	117
3.12 Operators with degenerating coefficients	120
3.13 A generalization of singular integral operators with continuous coefficients . .	123
3.14 Solution of Wiener-Hopf equations	128
3.15 Some applications	134
3.16 Exercises	151
Comments and references	154
4 Fredholm operators	157
4.1 Normally solvable operators	157
4.2 The restriction of normally solvable operators	157
4.3 Perturbation of normally solvable operators.	160
4.4 The normal solvability of the adjoint operator	161
4.5 Generalized invertible operators	162
4.6 Fredholm operators	165
4.7 Regularization of operators. Applications to singular integral operators . .	170
4.8 Index and trace	175
4.9 Functions of Fredholm operators and their index	178
4.10 The structure of the set of Fredholm operators	182
4.11 The Dependence of $\ker X$ and $\text{im} X$ on the operator X	187
4.12 The continuity of the function k_X	191
4.13 The case of a Hilbert space	195
4.14 The normal solvability of multiplication by a matrix function	199
4.15 Φ_{\pm} -operators	204
4.16 One-sided regularization of operators	212

4.17	Projections of invertible operators	218
4.18	Exercises	220
	Comments and references	220
5	Local Principles and their first applications	225
5.1	Localizing classes	225
5.2	Multipliers on \tilde{l}_p	230
5.3	paired equations with continuous coefficients on \tilde{l}_p	234
5.4	Operators of local type	242
5.5	Exercises	250
	Comments and references	252
	References	255
	Subject Index	265