Contents (Part II/A)

Preface	to Part II/A	vii
INTRO	ODUCTION TO THE SUBJECT	1
CHAP	ΓER 18	
Variati	onal Problems, the Ritz Method, and	
	a of Orthogonality	15
§18.1.	The Space $C_0^{\infty}(G)$ and the Variational Lemma	17
§18.1.		19
§18.2.		21
§18.4.	The Second and Third Boundary Value Problems and	
g10.4.	the Ritz Method	28
§18.5.	Eigenvalue Problems and the Ritz Method	32
§18.5.	The Hölder Inequality and its Applications	35
§18.0.	The History of the Dirichlet Principle and Monotone Operators	40
	The Main Theorem on Quadratic Minimum Problems	56
§18.8.	The Inequality of Poincaré-Friedrichs	59
§18.9.	The Functional Analytic Justification of the Dirichlet Principle	60
§18.10.	The Perpendicular Principle, the Riesz Theorem, and	
§18.11.	the Main Theorem on Linear Monotone Operators	64
010 13	The Extension Principle and the Completion Principle	70
§18.12.	Proper Subregions	71
§18.13.	The Smoothing Principle	72
§18.14.	The Idea of the Regularity of Generalized Solutions and	
§18.15.	the Lemma of Wayl	78
040.46	the Lemma of Weyl	79
§18.16.	The Localization Principle Convex Variational Problems, Elliptic Differential Equations,	
§18.17.	and Monotonicity	81
		xiii

xiv Contents (Part II/A)

§18.18.	The General Euler-Lagrange Equations	85
§18.19.	The Historical Development of the 19th and 20th Problems of	06
	Hilbert and Monotone Operators	86
§18.20.	Sufficient Conditions for Local and Global Minima and	93
	Locally Monotone Operators	93
CHAP'	ΓER 19	
	lerkin Method for Differential and Integral Equations,	
	edrichs Extension, and the Idea of Self-Adjointness	101
§19.1.	Elliptic Differential Equations and the Galerkin Method	108
§19.1.	Parabolic Differential Equations and the Galerkin Method	111
§19.2.	Hyperbolic Differential Equations and the Galerkin Method	113
§19.4.	Integral Equations and the Galerkin Method	115
§19.5.	Complete Orthonormal Systems and Abstract Fourier Series	116
§19.6.	Eigenvalues of Compact Symmetric Operators	
323.0.	(Hilbert-Schmidt Theory)	119
§19.7.	Proof of Theorem 19.B	121
§19.8.	Self-Adjoint Operators	124
§19.9.	The Friedrichs Extension of Symmetric Operators	126
§19.10.	Proof of Theorem 19.C	129
§19.11.	Application to the Poisson Equation	132
§19.12.	Application to the Eigenvalue Problem for the Laplace Equation	134
§19.13.	The Inequality of Poincaré and the Compactness	
•	Theorem of Rellich	135
§19.14.	Functions of Self-Adjoint Operators	138
§19.15.	Application to the Heat Equation	141
§19.16.	Application to the Wave Equation	143
§19.17.	Semigroups and Propagators, and Their Physical Relevance	145
§19.18.	Main Theorem on Abstract Linear Parabolic Equations	153
§19.19.	Proof of Theorem 19.D	155
§19.20.	Monotone Operators and the Main Theorem on	
	Linear Nonexpansive Semigroups	159
§19.21.	The Main Theorem on One-Parameter Unitary Groups	160
§19.22.	Proof of Theorem 19.E	162
§19.23.	Abstract Semilinear Hyperbolic Equations	164
§19.24.	Application to Semilinear Wave Equations	166
§19.25.	The Semilinear Schrödinger Equation	167
§19.26.	Abstract Semilinear Parabolic Equations, Fractional Powers of	160
040.05	Operators, and Abstract Sobolev Spaces	168
§19.27.	Application to Semilinear Parabolic Equations	171
§19.28.	Proof of Theorem 19.I	171
§19.29.	Five General Uniqueness Principles and Monotone Operators	174
§19.30.	A General Existence Principle and Linear Monotone Operators	175
CHAF	TER 20	
Differe	ence Methods and Stability	192
§20.1.	•	195
§20.2.	Approximation of Differential Quotients	199

Contents (Part II/A)		xv
§20.3.	Application to Boundary Value Problems for	
§20.5.	Ordinary Differential Equations	200
§20.4.	Application to Parabolic Differential Equations	203
§20.5.	Application to Elliptic Differential Equations	208
§20.6.	The Equivalence Between Stability and Convergence	210
§20.7.	The Equivalence Theorem of Lax for Evolution Equations	211
LINE	AR MONOTONE PROBLEMS	225
CHAP	TER 21	
Auxilia	ry Tools and the Convergence of the Galerkin	
Metho	d for Linear Operator Equations	229
821.1	Generalized Derivatives	231
§21.2.		235
§21.3.	The Soboley Embedding Theorems	237
§21.4.	Proof of the Sobolev Embedding Theorems	241
§21.5.	Duality in B-Spaces	251
§21.6.	Duality in H-Spaces	253
§21.7.	The Idea of Weak Convergence	255
§21.8.	The Idea of Weak* Convergence	260
§21.9.	Linear Operators	261
§21.10.	Bilinear Forms	262
§21.11.	Application to Embeddings	265 265
§21.12.	Projection Operators	203
§21.13.	Bases and Galerkin Schemes	271
§21.14.	Application to Finite Elements	275
§21.15.	Riesz-Schauder Theory and Abstract Fredholm Alternatives	213
§21.16.	The Main Theorem on the Approximation-Solvability of Linear	279
	Operator Equations, and the Convergence of the Galerkin Method	283
§21.17.	Interpolation Inequalities and a Convergence Trick	203
§21.18.	Application to the Refined Banach Fixed-Point Theorem and	285
	the Convergence of Iteration Methods	286
§21.19.	The Gagliardo-Nirenberg Inequalities	290
§21.20.	The Strategy of the Fourier Transform for Sobolev Spaces	292
§21.21.	Banach Algebras and Sobolev Spaces	294
§21.22.	Moser-Type Calculus Inequalities Weakly Sequentially Continuous Nonlinear Operators on	_, .
§21.23.	Sobolev Spaces	296
CIIAI	PTER 22	
CHAI	rt Space Methods and Linear Elliptic Differential Equations	314
	a t . 3 f ' D lalaman and tha	
§22.1.		320
000 0	Ritz Method Application to Boundary Value Problems	325
§22.2.	Duglity and a nosterior	
§22.3.	Error Estimates for the Ritz Method	335
822.4		337

xvi Contents (Part II/A)

§22.5.	Main Theorem on Linear Strongly Monotone Operators and	
	the Galerkin Method	339
§22.6.	Application to Boundary Value Problems	345
§22.7.	Compact Perturbations of Strongly Monotone Operators,	2.47
	Fredholm Alternatives, and the Galerkin Method	347
§22.8.	Application to Integral Equations	349
§22.9.	Application to Bilinear Forms	350
§22.10.	Application to Boundary Value Problems	351
§22.11.	Eigenvalue Problems and the Ritz Method	352
§22.12.	Application to Bilinear Forms	357
§22.13.	Application to Boundary-Eigenvalue Problems	361
§22.14.	Gårding Forms	364
§22.15.	The Gårding Inequality for Elliptic Equations	366
§22.16.	The Main Theorems on Gårding Forms	369
§22.17.	Application to Strongly Elliptic Differential Equations of Order 2m	371
§22.18.	Difference Approximations	374
§22.19.	Interior Regularity of Generalized Solutions	376
§22.20.	Proof of Theorem 22.H	378
§22.21.	Regularity of Generalized Solutions up to the Boundary	383
§22.22.	Proof of Theorem 22.I	384
СНАР	TER 23	
	Space Methods and Linear Parabolic Differential Equations	402
§23.1.	Particularities in the Treatment of Parabolic Equations	402
§23.2.	The Lebesgue Space $L_p(0, T; X)$ of Vector-Valued Functions	406
§23.3.	The Dual Space to $L_p(0, T; X)$	410
§23.4.	Evolution Triples	416
§23.5.	Generalized Derivatives	417
§23.6.	The Sobolev Space $W_p^1(0, T; V, H)$	422
§23.7.	Main Theorem on First-Order Linear Evolution Equations and	
325***	the Galerkin Method	423
§23.8.	Application to Parabolic Differential Equations	426
§23.9.	Proof of the Main Theorem	430
	TER 24	
	t Space Methods and Linear Hyperbolic	
Differe	ential Equations	452
§24.1.	Main Theorem on Second-Order Linear Evolution Equations	
	and the Galerkin Method	453
§24.2.	Application to Hyperbolic Differential Equations	456
824.3.	Proof of the Main Theorem	459