

Contents

Preface ix

List of Symbols xi

1. Background Material and Preliminaries

- 1.1 What Is Probability? 1
- 1.2 Random Variables and Measurability Results 5
- 1.3 Expectations and the Lebesgue Theory 10
- 1.4 Image Measure and the Fundamental Theorem
of Probability 18
Exercises 26

2. Independence and Strong Convergence

- 2.1 Independence 32
- 2.2 Convergence Concepts, Series, and Inequalities 44
- 2.3 Laws of Large Numbers 56
- 2.4 Applications to Empiric Distributions, Densities, Queuing, and
Random Walk 66
Exercises 84

3. Conditioning and Some Dependence Classes

- 3.1 Conditional Expectations 96
- 3.2 Conditional Probabilities 114
- 3.3 Markov Dependence 134
- 3.4 Existence of Various Random Families 152
- 3.5 Martingale Sequences 168
Exercises 197

4. Probability Distributions and Characteristic Functions

- 4.1 Distribution Functions and the Selection Principle 215

| | | | |
|--|--|-----|-----|
| 4.2 | Characteristic Functions, Inversion, and Lévy's Continuity Theorem | 227 | |
| 4.3 | Cramér's Theorem on Fourier Transforms of Signed Measures | 242 | |
| 4.4 | Bochner's Theorem on Positive Definite Functions | | 247 |
| 4.5 | Some Multidimensional Extensions | 257 | |
| 4.6 | Equivalence of Convergences for Sums of Independent Random Variables | 265 | |
| | Exercises | 268 | |
| 5. Weak Limit Laws | | | |
| 5.1 | Classical Central Limit Theorems | 281 | |
| 5.2 | Infinite Divisibility and the Lévy–Khintchine Formula | | 294 |
| 5.3 | General Limit Laws, Including Stability | 309 | |
| 5.4 | Invariance Principles | 332 | |
| 5.5 | Kolmogorov's Law of the Iterated Logarithm | | 356 |
| | Exercises | 368 | |
| 6. Application to Some Statistical Estimation Problems | | | |
| 6.1 | Estimation of Parameters in Stochastic Difference Equations | 388 | |
| 6.2 | Continuation: Limit Distributions | 397 | |
| 6.3 | Higher-Order Schemes: Difficulties and Unsettled Problems | 407 | |
| | Exercises | 409 | |
| 7. Stopping Times, Martingales, and Convergence | | | |
| 7.1 | Stopping Times and Their Calculus | 412 | |
| 7.2 | Wald's Equation and an Application | 416 | |
| 7.3 | Stopped Martingales | 421 | |
| 7.4 | Asymptotic Martingales | 427 | |
| 7.5 | Improved Convergence | 431 | |
| | Exercises | 433 | |
| 8. Limit Laws for Some Dependent Sequences | | | |
| 8.1 | Central Limit Theorems | 435 | |
| 8.2 | Limit Laws for a Random Number of Random Variables | | 442 |
| 8.3 | Ergodic Sequences | 456 | |
| | Exercises | 461 | |

9. A Glimpse into Stochastic Processes

| | |
|---|-----|
| 9.1 Brownian Motion: Definition and Construction | 464 |
| 9.2 Some Properties of Brownian Motion | 468 |
| 9.3 Law of the Iterated Logarithm for Brownian Motion | 472 |
| 9.4 Gaussian and Additive Processes | 476 |
| 9.5 Second-Order Processes | 480 |
| Exercises | 483 |
| References | 485 |
| Author Index | 489 |
| Subject Index | 492 |