

# Contents

<b>Preface</b>	<b>iii</b>
<b>1 Basic Concepts</b>	<b>1</b>
1.1 The Scope and Methods of Probability Theory	1
1.2 Experiments and Events	4
1.3 The Axioms of Probability	9
1.4 Elementary Consequences of the Axioms	12
1.5 Classical Probability Spaces	15
1.6 Selection without Replacement: The Case of Two Types of Items	21
1.7 Proof of Theorem 1.8	26
1.8 Selection with Replacement from a Lot Containing Two Types of Items	27
1.9 Further Models with Classical Probability Spaces	30
1.10 Finite and Denumerable Sample Spaces	33
1.11 Geometric Probabilities	35
1.12 Exercises	38
<b>2 Conditional Probability and Independence</b>	<b>43</b>
2.1 The Definition of Conditional Probability	43
2.2 The Multiplication Rule for Intersections	46
2.3 The Total Probability Rule and Bayes' Theorem	47
2.4 The Independence of Two Events	51

2.5	Extensions of the Concept of Independence	54
2.6	A Return to the Relation of the Relative Frequency and Probability	58
2.7	The Standard Normal Distribution Function	65
2.8	The Proof of the Chebyshev Inequality	68
2.9	The Proof of Theorem 2.10	69
2.10	The Poisson Approximation to the Binomial Probabilities	74
2.11	Exercises	76
<b>3</b>	<b>Random Variables</b>	<b>80</b>
3.1	The Concept of Random Variables	80
3.2	Properties of Distribution Functions	82
3.3	The Exponential Distribution As Life Distribution	86
3.4	Discrete Random Variables	88
3.5	Absolutely Continuous Distribution Functions	93
3.6	The Normal Distribution	96
3.7	Multivariate Distributions	99
3.8	Conditional Densities	106
3.9	Independence of Random Variables	109
3.10	Sums of Independent Random Variables	112
3.11	Methods for Determining Distribution Functions	116
3.12	Exercises	121
<b>4</b>	<b>Expectation and Variance</b>	<b>126</b>
4.1	The Expected Value of Discrete Random Variable	126
4.2	The Expectation of Absolutely Continuous Random Variable	134
4.3	The Distribution and the Expectation of a Function of Random Variable	138
4.4	The Variance and the Correlation Coefficient	143
4.5	The Chebyshev Inequality and the (Weak) Law of Large Numbers	149
4.6	Exercises	152
<b>5</b>	<b>Limit Theorems</b>	<b>155</b>
5.1	The Central Limit Theorem	155
5.2	Moment Generating Functions, and the Proof of the Central Limit Theorem	161
5.3	Asymptotic Extreme Value Distributions	164
5.4	Two Exact Models Via Limit Theorems	168
5.5	Sums of Indicator Variables	170
5.6	Exercises	175

<b>6</b>	<b>Miscellaneous Topics</b>	<b>177</b>
6.1	Waiting-Time Paradoxes	177
6.2	An Unexpected Situation Concerning Independent Random Variables	182
6.3	Once More on Independence	184
6.4	The Mean and the Standard Deviation for Normal Samples	185
6.5	The Borel-Cantelli Lemma; the Strong Convergence of the Relative Frequency	187
6.6	Fair Games	190
	<b>Index</b>	<b>197</b>