

Contents

<i>Preface</i>	vii
----------------------	-----

CHAPTER 1 Probability Spaces

1.1 Sigma Fields	1
1.2 Probability Measures	5
1.3 Random Variables	9

CHAPTER 2 Probability Distributions

2.1 Univariate Distribution Functions	15
2.2 Multivariate Distribution Functions	23
2.3 Distribution of a Set of Infinitely Many Random Variables	29
2.4 Expectation	34
2.5 Characteristic Functions	41

CHAPTER 3 Stochastic Independence

3.1 Independent Events	57
3.2 Independent Random Variables	61
3.3 The Zero-One Law	70

CHAPTER 4 Basic Limiting Operations

4.1	Convergence of Distribution Functions	77
4.2	The Continuity Theorem	88
4.3	Refinements of the Continuity Theorem for Nonvanishing Characteristic Functions	92
4.4	The Four Types of Convergence: Almost Sure, in Law, in Probability, and in r th Mean	99

CHAPTER 5 Strong Limit Theorems for Independent Random Variables

5.1	Almost Sure Convergence of Series of Independent Random Variables	107
5.2	Proof that Convergence in Law of a Series of Independent Random Variables Implies Almost Sure Convergence	115
5.3	The Strong Law of Large Numbers	122
5.4	The Glivenko-Cantelli Theorem	126
5.5	Inequalities for the Law of the Iterated Logarithm	129
5.6	The Law of the Iterated Logarithm	137

CHAPTER 6 The Central Limit Theorem

6.1	Infinitely Divisible Distributions	147
6.2	Canonical Representation of Infinitely Divisible Characteristic Functions	153
6.3	Convergence of Infinitely Divisible Distribution Functions	161
6.4	Infinitesimal Systems of Random Variables	167
6.5	The General Limit Theorem for Sequences of Sums of Independent Random Variables	177
6.6	Convergence to the Normal and Poisson Distributions	194

CHAPTER 7 Conditional Expectation and Martingale Theory

7.1	Conditional Expectation	209
7.2	Martingales and Submartingales	220
7.3	Martingale and Submartingale Convergence Theorems	229
7.4	Brownian Motion	239

CHAPTER 8 An Introduction to Stochastic Processes and, in Particular, Brownian Motion

8.1	Probability Measures over Function Spaces	245
8.2	Separable Stochastic Processes	249

8.3	Continuity and Nonrectifiability of Almost All Sample Functions of Separable Brownian Motion	254
8.4	The Law of the Iterated Logarithm for Separable Brownian Motion	265
	Suggested Reading	270
	<i>Index</i>	271