# Contents

Introduction

A Useful Notation

<table>
<thead>
<tr>
<th>Part 1 Operational Foundations</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Decision-making and Probability</td>
<td></td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>1.2 The exploration of oil deposits</td>
<td>4</td>
</tr>
<tr>
<td>1.3 An experiment: forecasting the result of football matches</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Responses to test items</td>
<td>8</td>
</tr>
<tr>
<td>1.5 Probability theory deduced from the notion of admissible decision</td>
<td>9</td>
</tr>
<tr>
<td>1.6 The complete construction of probability theory</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part 2 Conditional Probabilities and Decision Theory</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Introduction</td>
<td>13</td>
</tr>
<tr>
<td>2.2 The case of conditional probabilities</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Proof</td>
<td>15</td>
</tr>
<tr>
<td>2.4 The lines of uniform improvement</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part 3 Does it Make Sense to Speak of “Good Probability Appraisers”?</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part 4 Methods for Discriminating Levels of Partial Knowledge Concerning a Test Item</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Introduction</td>
<td>25</td>
</tr>
<tr>
<td>4.2 How to make use of decision theory and why</td>
<td>28</td>
</tr>
<tr>
<td>4.3 The continuous method</td>
<td>29</td>
</tr>
<tr>
<td>4.4 Some practicable methods: (A) purely rank-order methods</td>
<td>32</td>
</tr>
<tr>
<td>4.5 Some practicable methods: (B) flexible schemes with two or three permitted levels of response</td>
<td>34</td>
</tr>
<tr>
<td>4.6 Further approaches: (C) strict least distance methods</td>
<td>39</td>
</tr>
</tbody>
</table>
CONTENTS

4.7 Some more methods: (D) least distance methods (flexible) 47
4.8 Aspects deserving further discussion 50
4.9 How to choose a suitable method 53
4.10 On some factors influencing information 57
4.11 How to operate appropriately in the long run 60
4.12 Some other problems 61

Part 2 Axiomatic Foundations

5 On the Axiomatization of Probability Theory 65
  5.1 Introduction 67
  5.2 Outline of chapter 68
  5.3 The axioms of Kolmogorov 68
  5.4 Topics for discussion 69
  5.5 Events and elementary cases 69
  5.6 Domains of events and probabilities 72
  5.7 On different approaches to probability 73
  5.8 Some consequences of the formal approach 74
  5.9 Finite additivity 75
  5.10 The domain of uniqueness of the extension 79
  5.11 Geometrical interpretations 80
  5.12 The definition of conditional probability 81
  5.13 An analytical interpretation 82
  5.14 The basic problem 83
  5.15 Complete additivity 84
  5.16 Some observations 85
  5.17 Some examples 86
  5.18 Preliminaries 86
  5.19 Complete additivity and the asymptotic approach 89
  5.20 Complete additivity and the subjectivistic approach 90
  5.21 Complete additivity and perfect additivity 92
  5.22 Closure and the different approaches to probability 93
  5.23 Reasons in favour of closure 95
  5.24 The continuity postulate 96
  5.25 Some remarks 97
  5.26 The paradox of non-conglomerability 98
  5.27 Continuity with respect to conditioning 100
  5.28 The case of hypotheses of zero probability 100
  5.29 Some observations on probabilities defined through limiting processes 101
  5.30 Examples of non-conglomerability 102
  5.31 Critical remarks on the notion of conditional probability 104
CONTENTS

5.32 Toward a general formulation 105
5.33 Random quantities and complete additivity 105
5.34 Continuity on linear spaces 107
5.35 The continuous extension 108
5.36 An outline of an abstract theory of integration 110
5.37 Some additional problems 111

6 On the Abstract Theory of Measure and Integration 115
  6.1 Introduction 115
  6.2 Preliminaries 116
  6.3 Measure and integral 117
  6.4 The axioms 118
  6.5 Extension of measures 119
  6.6 Further problems 120
  6.7 Compatible measures 121
  6.8 Invariant measures 121
  6.9 Complete additivity and continuity 122
  6.10 Continuity on the entire $\mathcal{F}$ 124
  6.11 Closure of a set with respect to a linear space 124
  6.12 Further remarks on continuity 125
  6.13 Special methods of extension 126
  6.14 Further developments 127

7 The Structure of Distributions on Abstract Spaces 129
  7.1 Preliminaries 129
  7.2 Discrete masses 131
  7.3 Agglutinated masses 131
  7.4 Continuous masses 133
  7.5 "Measures" and condensed masses 134
  7.6 Diffuse masses ("absolutely continuous" with respect to $m$) 135
  7.7 Approximation by means of masses with finite "density" 137
  7.8 Conclusions 139

Part 3 Induction and Statistics 141

8 How to Choose the Initial Probabilities 143

9 Probability, Statistics and Induction: Their Relationship According to the Various Points of View 147
  9.1 Introduction 147
  9.2 Historical and comparative survey: from the beginnings to the crisis of the classical formulation 157
9.3 The rise of objectivistic concepts 162  
9.4 The erosion of the objectivistic positions 175  
9.5 Critical examination of controversial aspects 187  
9.6 Reconstruction of the classical formulation according to the subjectivistic viewpoint 209  

10 On the Notion of Partial Exchangeability 229  
  10.1 Introduction 229  
  10.2 Example with three subgroups 231  
  10.3 Example with four subgroups 243  

Bibliography 247  
Index 259