

Contents



Preface xiii

1. Introduction to Probability Theory 1

- 1.1. Introduction 1
- 1.2. Sample Space and Events 1
- 1.3. Probabilities Defined on Events 4
- 1.4. Conditional Probabilities 7
- 1.5. Independent Events 10
- 1.6. Bayes' Formula 12
 - Exercises 15
 - References 21

2. Random Variables 23

- 2.1. Random Variables 23
- 2.2. Discrete Random Variables 27
 - 2.2.1. The Bernoulli Random Variable 28
 - 2.2.2. The Binomial Random Variable 29
 - 2.2.3. The Geometric Random Variable 31
 - 2.2.4. The Poisson Random Variable 32
- 2.3. Continuous Random Variables 34
 - 2.3.1. The Uniform Random Variable 35
 - 2.3.2. Exponential Random Variables 36
 - 2.3.3. Gamma Random Variables 37
 - 2.3.4. Normal Random Variables 37

- 2.4. Expectation of a Random Variable 38
 - 2.4.1. The Discrete Case 38
 - 2.4.2. The Continuous Case 41
 - 2.4.3. Expectation of a Function of a Random Variable 43
- 2.5. Jointly Distributed Random Variables 47
 - 2.5.1. Joint Distribution Functions 47
 - 2.5.2. Independent Random Variables 51
 - 2.5.3. Covariance and Variance of Sums of Random Variables 53
 - 2.5.4. Joint Probability Distribution of Functions of Random Variables 61
- 2.6. Moment Generating Functions 64
 - 2.6.1. The Joint Distribution of the Sample Mean and Sample Variance from a Normal Population 74
- 2.7. Limit Theorems 77
- 2.8. Stochastic Processes 83
 - Exercises 85
 - References 96

3. Conditional Probability and Conditional Expectation 97

- 3.1. Introduction 97
- 3.2. The Discrete Case 97
- 3.3. The Continuous Case 102
- 3.4. Computing Expectations by Conditioning 105
 - 3.4.1. Computing Variances by Conditioning 116
- 3.5. Computing Probabilities by Conditioning 119
- 3.6. Some Applications 136
 - 3.6.1. A List Model 136
 - 3.6.2. A Random Graph 138
 - 3.6.3. Uniform Priors, Polya's Urn Model, and Bose–Einstein Statistics 146
 - 3.6.4. Mean Time for Patterns 150
 - 3.6.5. A Compound Poisson Identity 154
 - 3.6.6. The k -Record Values of Discrete Random Variables 158
- Exercises 161

4. Markov Chains 181

- 4.1. Introduction 181
- 4.2. Chapman–Kolmogorov Equations 185
- 4.3. Classification of States 189

- 4.4. Limiting Probabilities 200
- 4.5. Some Applications 213
 - 4.5.1. The Gambler's Ruin Problem 213
 - 4.5.2. A Model for Algorithmic Efficiency 217
 - 4.5.3. Using a Random Walk to Analyze a Probabilistic Algorithm for the Satisfiability Problem 220
- 4.6. Mean Time Spent in Transient States 226
- 4.7. Branching Processes 228
- 4.8. Time Reversible Markov Chains 232
- 4.9. Markov Chain Monte Carlo Methods 243
- 4.10. Markov Decision Processes 248
 - Exercises 252
 - References 268

5. The Exponential Distribution and the Poisson Process 269

- 5.1. Introduction 269
- 5.2. The Exponential Distribution 270
 - 5.2.1. Definition 270
 - 5.2.2. Properties of the Exponential Distribution 272
 - 5.2.3. Further Properties of the Exponential Distribution 279
 - 5.2.4. Convolutions of Exponential Random Variables 284
- 5.3. The Poisson Process 288
 - 5.3.1. Counting Processes 288
 - 5.3.2. Definition of the Poisson Process 289
 - 5.3.3. Interarrival and Waiting Time Distributions 293
 - 5.3.4. Further Properties of Poisson Processes 295
 - 5.3.5. Conditional Distribution of the Arrival Times 301
 - 5.3.6. Estimating Software Reliability 313
- 5.4. Generalizations of the Poisson Process 316
 - 5.4.1. Nonhomogeneous Poisson Process 316
 - 5.4.2. Compound Poisson Process 321
 - 5.4.3. Conditional or Mixed Poisson Processes 327
 - Exercises 330
 - References 348

6. Continuous-Time Markov Chains 349

- 6.1. Introduction 349
- 6.2. Continuous-Time Markov Chains 350
- 6.3. Birth and Death Processes 352

- 6.4. The Transition Probability Function $P_{ij}(t)$ 359
- 6.5. Limiting Probabilities 368
- 6.6. Time Reversibility 376
- 6.7. Uniformization 384
- 6.8. Computing the Transition Probabilities 388
 - Exercises 390
 - References 399

7. Renewal Theory and Its Applications 401

- 7.1. Introduction 401
- 7.2. Distribution of $N(t)$ 403
- 7.3. Limit Theorems and Their Applications 407
- 7.4. Renewal Reward Processes 416
- 7.5. Regenerative Processes 425
 - 7.5.1. Alternating Renewal Processes 428
- 7.6. Semi-Markov Processes 434
- 7.7. The Inspection Paradox 437
- 7.8. Computing the Renewal Function 440
- 7.9. Applications to Patterns 443
 - 7.9.1. Patterns of Discrete Random Variables 443
 - 7.9.2. The Expected Time to a Maximal Run of Distinct Values 451
 - 7.9.3. Increasing Runs of Continuous Random Variables 453
- 7.10. The Insurance Ruin Problem 455
 - Exercises 460
 - References 472

8. Queueing Theory 475

- 8.1. Introduction 475
- 8.2. Preliminaries 476
 - 8.2.1. Cost Equations 477
 - 8.2.2. Steady-State Probabilities 478
- 8.3. Exponential Models 480
 - 8.3.1. A Single-Server Exponential Queueing System 480
 - 8.3.2. A Single-Server Exponential Queueing System Having Finite Capacity 487
 - 8.3.3. A Shoeshine Shop 490
 - 8.3.4. A Queueing System with Bulk Service 493
- 8.4. Network of Queues 496
 - 8.4.1. Open Systems 496
 - 8.4.2. Closed Systems 501

- 8.5. The System $M/G/1$ 507
 - 8.5.1. Preliminaries: Work and Another Cost Identity 507
 - 8.5.2. Application of Work to $M/G/1$ 508
 - 8.5.3. Busy Periods 509
- 8.6. Variations on the $M/G/1$ 510
 - 8.6.1. The $M/G/1$ with Random-Sized Batch Arrivals 510
 - 8.6.2. Priority Queues 512
 - 8.6.3. An $M/G/1$ Optimization Example 515
- 8.7. The Model $G/M/1$ 519
 - 8.7.1. The $G/M/1$ Busy and Idle Periods 524
- 8.8. A Finite Source Model 525
- 8.9. Multiserver Queues 528
 - 8.9.1. Erlang's Loss System 529
 - 8.9.2. The $M/M/k$ Queue 530
 - 8.9.3. The $G/M/k$ Queue 530
 - 8.9.4. The $M/G/k$ Queue 532
- Exercises 534
- References 546

9. Reliability Theory 547

- 9.1. Introduction 547
- 9.2. Structure Functions 547
 - 9.2.1. Minimal Path and Minimal Cut Sets 550
- 9.3. Reliability of Systems of Independent Components 554
- 9.4. Bounds on the Reliability Function 559
 - 9.4.1. Method of Inclusion and Exclusion 560
 - 9.4.2. Second Method for Obtaining Bounds on $r(\mathbf{p})$ 569
- 9.5. System Life as a Function of Component Lives 571
- 9.6. Expected System Lifetime 580
 - 9.6.1. An Upper Bound on the Expected Life of a Parallel System 584
- 9.7. Systems with Repair 586
 - 9.7.1. A Series Model with Suspended Animation 591
- Exercises 593
- References 600

10. Brownian Motion and Stationary Processes 601

- 10.1. Brownian Motion 601
- 10.2. Hitting Times, Maximum Variable, and the Gambler's Ruin Problem 605

- 10.3. Variations on Brownian Motion 607
 - 10.3.1. Brownian Motion with Drift 607
 - 10.3.2. Geometric Brownian Motion 607
- 10.4. Pricing Stock Options 608
 - 10.4.1. An Example in Options Pricing 608
 - 10.4.2. The Arbitrage Theorem 611
 - 10.4.3. The Black–Scholes Option Pricing Formula 614
- 10.5. White Noise 620
- 10.6. Gaussian Processes 622
- 10.7. Stationary and Weakly Stationary Processes 625
- 10.8. Harmonic Analysis of Weakly Stationary Processes 630
 - Exercises 633
 - References 638

11. Simulation 639

- 11.1. Introduction 639
- 11.2. General Techniques for Simulating Continuous Random Variables 644
 - 11.2.1. The Inverse Transformation Method 644
 - 11.2.2. The Rejection Method 645
 - 11.2.3. The Hazard Rate Method 649
- 11.3. Special Techniques for Simulating Continuous Random Variables 653
 - 11.3.1. The Normal Distribution 653
 - 11.3.2. The Gamma Distribution 656
 - 11.3.3. The Chi-Squared Distribution 657
 - 11.3.4. The Beta (n, m) Distribution 657
 - 11.3.5. The Exponential Distribution—The Von Neumann Algorithm 658
- 11.4. Simulating from Discrete Distributions 661
 - 11.4.1. The Alias Method 664
- 11.5. Stochastic Processes 668
 - 11.5.1. Simulating a Nonhomogeneous Poisson Process 669
 - 11.5.2. Simulating a Two-Dimensional Poisson Process 676
- 11.6. Variance Reduction Techniques 679
 - 11.6.1. Use of Antithetic Variables 680
 - 11.6.2. Variance Reduction by Conditioning 684
 - 11.6.3. Control Variates 688
 - 11.6.4. Importance Sampling 690
- 11.7. Determining the Number of Runs 696

11.8. Coupling from the Past 696
Exercises 699
References 707

Appendix: Solutions to Starred Exercises 709

Index 749