

Contents

1 The Basic Probability Model 1

<i>Introduction</i>	1
<i>Fair Coin-tossing</i>	1
<i>Models with a Finite Number of Outcomes</i>	2
<i>The Heuristics of Model-building</i>	3
<i>The Additivity of Probability</i>	4
<i>Infinite Outcomes Spaces</i>	7
<i>Probabilities as Integrals</i>	14
<i>Summary</i>	18

2 Some Classical Models 19

<i>Introduction</i>	19
<i>Equally Likely Outcomes and Independent Trials</i>	19
<i>The Binomial Distribution</i>	22
<i>The Hypergeometric Distribution</i>	27
<i>The Multinomial Distribution</i>	30
<i>The Poisson Distribution</i>	32
<i>The Exponential Distribution</i>	37
<i>The Uniform Distribution</i>	40
<i>Summary</i>	42

3 Random Variables 44

<i>The Definitions</i>	44
<i>Remarks on Random Variables</i>	49
<i>The Three Types of Distributions of Random Variables</i>	50
<i>The Cumulative Distribution Function</i>	53
<i>Densities of Functions of Random Variables</i>	55
<i>The Expected Value of a Random Variable</i>	56
<i>Expectation of a Function of a Random Variable</i>	61
<i>The Joint Distribution of Two Random Variables</i>	65
<i>Expectations of Functions of Two Random Variables</i>	70
<i>Covariance and the Correlation Coefficient</i>	74
<i>Many Random Variables</i>	76
<i>An Infinite Number of Random Variables</i>	79
<i>Summary</i>	82

4 Independent Random Variables 84

<i>Definitions</i>	84
<i>Independence is a Family Property</i>	89
<i>More on the Poisson Process</i>	93
<i>Independence, Expectations, and Variances</i>	98
<i>Identically Distributed Random Variables, and the Law of Averages</i>	102
<i>Sums of Small Independent Components are Normally Distributed</i>	106
<i>How Deviant is a Deviation?</i>	112
<i>How Good is the Normal Approximation?</i>	117
<i>The Binomial Distribution</i>	119
<i>The Poisson Distribution</i>	120
<i>The Exponential Distribution</i>	122
<i>The Uniform Distribution</i>	123
<i>Summary</i>	124

5 Conditional Probability 126

- Introduction* 126
- Some Examples* 130
- The Distribution of X Given $Y = y$* 134
- A Useful Rule* 137
- The Addition Rule for Conditional Probabilities* 138
- Conditional Expectation* 143
- Dependent Sequences of Random Variables* 147
- Summary* 151

6 Markov Chains 152

- Definitions* 152
- Some Examples* 155
- Some General Properties of Markov Motion* 164
- The Stability of a Markov System* 168
- Integer States and Recurrence Times* 175
- The Stability Problem Solved for Integer States* 180
- The Difference Equation Method* 188
- Summary* 194

7 Continuous Time Markov Processes 196

- Models for Continuous Time Processes* 196
- Continuous Time Integer-valued Markov Processes* 197
- The Infinitesimal Transition Scheme* 200
- The Differential Equations for the Transition Probabilities* 205
- The Steady-state Distributions* 208
- How Does a Markov Process Operate?* 211
- The Difference Equation* 215
- Summary* 216

8 Vector Independence and the Multivariate Normal Distribution 217

Introduction 217

The Covariance Matrix and Means Vector 218

Independence of Random Vectors 221

Sums of Vector Variables 224

The Central Limit Theorem 226

The Multivariate Normal Distribution 233

Properties of the Multivariate Normal Distribution 242

Gaussian Processes 246

Summary 247

9 Stationary Time Series 249

Introduction 249

Gaussian and Second-order Stationary Processes 253

Almost Periodic Processes 257

The General Frequency Representation 264

Linear Systems 274

White Noise and the Gauss-Markov Process 281

The General Prediction Problem 292

Linear Prediction and Filtering 300

Modeling and Ergodicity 313

Summary 318

Table 320

Index 321