Contents

Contents

	Preface
	1 Introduction 15
.2 .3 .4	A few ideas and examples 15 The empirical basis 16 Averages over a finite population 19 Repeated experiments; expectation 21 More on sample spaces and observables 22
	2 Expectation 25
2.2 2.3 2.4 2.5 2.6	Random variables 25 Axioms for the expectation operator 26 Events; probability 28 Some examples of an expectation 29 Applications: optimization problems 33 Applications: least square approximation of random variables 35 Some implications of the axioms 38
	3 Probability 43
3.2 3.3 3.4	Probability measure 43 Probability and expectation 45 Expectation as an integral 48 Elementary properties of events and probabilities 50 Fields of events 53
	4 Some Simple Processes 56
1.2 1.3 1.4 1.5 1.6	Equiprobable sample spaces 56 A coin-tossing example; stochastic convergence 57 A more general example; the binomial distribution 60 Multiple classification; the multinomial distribution 63 Sampling without replacement; the hypergeometric distribution 65 Indefinite sampling; the geometric and negative binomial distributions 68 Sampling from a continuum; the Poisson process 71 'Nearest neighbours'; the exponential and gamma distributions 73 Other simple processes 74

5	Cond	ition	ing	80
,	COIIG	11101	u5	00

- 5.1 Conditional expectation 80
- 5.2 Conditional probability 84
- 5.3 A conditional expectation as a random variable 88
- 5.4 Conditioning on a σ -field of events 90
- 5.5 Statistical independence 93
- 5.6 Elementary consequences of independence 95
- 5.7 Partial independence; orthogonality 101

6 Applications of the Independence Concept 104

- 6.1 Mean square convergence of sample averages 104
- 6.2 Convergence of sample averages; some stronger results 106
- 6.3 Renewal processes 109
- 6.4 Recurrent states (events) 113
- 6.5 A result in statistical mechanics 117
- 6.6 Branching processes 120

7 Markov Processes 128

- 7.1 The Markov property 128
- 7.2 Some particular Markov processes 133
- 7.3 The simple random walk 137
- 7.4 Markov processes in continuous time 139
- 7.5 The Poisson process in time 141
- 7.6 Birth processes 144
- 7.7 Birth and death processes 147

8 Continuous Distributions 150

- 8.1 Distributions with a density 150
- 8.2 Functions of random variables 153
- 8.3 Conditional densities 159
- 8.4 Characteristic functions 161
- 8.5 The normal distribution; normal convergence 167
- 8.6 A direct proof of normal convergence 171

9 Convergence of Random Sequences 174

- 9.1 Characterization of convergence 174
- 9.2 Types of convergence 176
- 9.3 Some consequences 178
- 9.4 Kolmogorov's inequality, and refinements of it 180
- 9.5 The laws of large numbers 183
- 9.6 Martingale convergence, and applications 186
- 9.7 Convergence in rth mean 189

10 Extension 194

- 10.1 Extension of the expectation functional: the finite case 194
 10.2 Generalities on the infinite case 197
- 10.3 Extension on a linear lattice 199
- 10.4 Extension on a quadratic field 201

11 Examples of Extension 204

- 11.1 Integrable functions of a scalar random variable 204
- 11.2 Expectations derivable from the characteristic function 206

12 Some Interesting Processes 212

- 12.1 Quantum mechanics 212
- 12.2 Information theory 218
- 12.3 Dynamic programming; stock control 223
- 12.4 Stochastic differential equations; generalized processes 227

References 233

Index 235