Contents

1	Events and their probabilities	
1.1	Introduction	
1.2	Events as sets	
1.3	Probability	
1.4	Conditional probability: a fundamental lemma	:
1.5	Independence	1.
1.6	Completeness and product spaces	14
1.7	Worked examples	10
1.8	Problems	2
2	Random variables and their distributions	
2.1	Random variables	25
2.2	The law of averages	29
2.3	Discrete and continuous variables	32
2.4	Worked examples	35
2.5	Random vectors	38
2.6	Monte Carlo simulation	41
2.7	Problems	43
3	Discrete random variables	
3.1	Probability mass functions	46
3.2	Independence	47
3.3	Expectation	50
3.4	Indicators and matching	56
3.5	Examples of discrete variables	60
3.6	Dependence	62
3.7	Conditional distributions and conditional expectation	67
3.8	Sums of random variables	70
3.9	Simple random walk	71
3.10	Random walk: counting sample paths	75
3.11	Problems	83
4	Continuous random variables	
4.1	Probability density functions	89
4.2	Independence	91
4.3	Expectation	92
4.4	Examples of continuous variables	94
4.5	Dependence	98
4.6	Conditional distributions and conditional expectation	103
4.7	Functions of random variables	107

	X	Contents
4.8	Sums of random variables	113
4.9	Multivariate normal distribution	114
4.10	Distributions arising from the normal distribution	118
4.11	Problems	121
5	Generating functions and their applications	
5.1	Generating functions	127
5.2	Some applications	135
5.3	Random walk	141
5.4	Branching processes	150
5.5	Age-dependent branching processes	155
5.6	Expectation revisited	158
5.7	Characteristic functions	162
5.8	Examples of characteristic functions	167
5.9	Inversion and continuity theorems	170
5.10	Two limit theorems	174
5.11	Large deviations	183
5.12	Problems	187
6	Markov chains	
6.1	Markov processes	194
6.2	Classification of states	201
6.3	Classification of chains	204
6.4	Stationary distributions and the limit theorem	207
6.5	Time-reversibility	218
6.6	Chains with finitely many states	221
6.7	Branching processes revisited	224
6.8	Birth processes and the Poisson process	228
6.9	Continuous-time Markov chains	239
6.10 6.11	Uniform semigroups	246
6.12	Birth-death processes and imbedding	249
6.13	Special processes Problems	256
0.13	Frooletiis	264
7	Convergence of random variables	
7.1	Introduction	271
7.2	Modes of convergence	274
7.3	Some ancillary results	285
7.4	Laws of large numbers	293
7.5	The strong law	293 297
7.6	The law of the iterated logarithm	301
7.7	Martingales	302
7.8	Martingale convergence theorem	302
7.9	Prediction and conditional expectation	314
7.10	Uniform integrability	322
7.11	Problems	326
		240

	Contents	xi
8	Random processes	
8.1	Introduction	332
8.2	Stationary processes	333
8.3	Renewal processes	337
8.4	Queues	340
8.5	The Wiener process	342
8.6	What is in a name?	343
8.7	Problems	346
9	Stationary processes	
9.1	Introduction	347
9.2	Linear prediction	349
9.3	Autocovariances and spectra	352
9.4	Stochastic integration and the spectral representation	360
9.5	The ergodic theorem	367
9.6	Gaussian processes	380
9.7	Problems	384
· · ·	rootems	304
10	Renewals	
10.1	The renewal equation	388
10.2	Limit theorems	393
10.3	Excess life	398
10.4	Applications	401
10.5	Problems	410
11	Queues	
11.1	Single-server queues	414
11.2	M/M/1	416
11.3	M/G/1	420
11.4	G/M/1	427
11.5	G/G/1	431
11.6	Heavy traffic	438
11.7	Problems	439
12	Martingales	
12.1	Introduction	443
12.2	Martingale differences and Hoeffding's inequality	448
12.3	Crossings and convergence	453
12.4	Stopping times	459
12.5	Optional stopping	464
12.6	The maximal inequality	469
12.7	Backward martingales and continuous-time martingales	472
12.8	Some examples	477
12.9	Problems	482

ţ

	xii		Contents	
13	Diffus	ion processes		
13.1	Introdu		487	
13.2	- 210 William Motion			
13.3				
13.4	13.4 First passage times			
13.5				
13.6	Excursions, and the Brownian bridge			
13.7		al theory	509 512	
13.8	Problen	ns	518	
Appen	dix I.	Foundations and notation	521	
Appendix II.		Further reading	526	
Appendix III.		History and varieties of probability	527	
	Appendix IV. John Arbuthnot's Preface to Of the laws of			
chance (1692)			529	
Bibliography				
List of notation				
Index				