Contents

Preface iii

1. REVIEW OF BASIC CONCEPTS 1

   1.1 Events 1
   1.2 Probability 5
   1.3 A method of constructing probabilities 12
   1.4 Classical probability spaces 19
   1.5 Simple random variables 20
   1.6 Conditional probability: The elementary case 26
   1.7 Independence of events 28
   1.8 Random variables, distribution functions 33
   References 38

2. EXPECTATION AND INTEGRAL, WEAK AND STRONG CONVERGENCE 41

   2.1 Definitions of expectation and integral 41
   2.2 Basic properties of expectation 45
   2.3 The inequalities of Markov, Chebyshev, and Kolmogorov 48
   2.4 Sequences of integrals 54
   2.5 The strong law of large numbers: The proof of Theorem 1 57
   2.6 Zero-one laws 62
   2.7 Lebesgue–Stieltjes integrals 66
   2.8 Weak convergence 76
   References 82

3. TRANSFORMS OF DISTRIBUTION 85

3.1 Characteristic functions: Basic properties 85
3.2 Characteristic functions: The uniqueness and continuity theorems 93
3.3 Classical forms of the central limit theorem, and a model for measurement errors 106
3.4 Characteristic functions: Inequalities 115
3.5 Multivariate characteristic functions 122
3.6 Laplace transforms 124
3.7 Generating functions 128
References 132

4. INFINITE SEQUENCES OF INDEPENDENT RANDOM VARIABLES: WEAK CONVERGENCE 135

4.1 Complete convergence of sums without normalization, infinite series 135
4.2 Decomposition of the normal distribution 146
4.3 Lévy's metric 150
4.4 Zolotarev's theorem on asymptotic normality, the theorem of Lindeberg and Feller 157
4.5 Speed of convergence: The Berry–Esseen theorem 165
4.6 The class L of limiting distributions 171
References 182

5. TRIANGULAR ARRAYS OF INDEPENDENT RANDOM VARIABLES, INFINITELY DIVISIBLE DISTRIBUTIONS 185

5.1 Introduction 185
5.2 More on infinitely divisible distributions 189
5.3 Convergence under UAN 196
5.4 Convergence to special distributions 204
References 216

6. INDEPENDENT AND IDENTICALLY DISTRIBUTED RANDOM VARIABLES 219

6.1 Cauchy's functional equation 219
6.2 Stable distributions 224
6.3 Regularly varying functions 232
6.4 Domains of attraction for stable distributions 238
6.5 The asymptotic theory of the extremes 244

6.6 The law of the iterated logarithm 253
References 260

7. CONDITIONAL EXPECTATION; MARTINGALES 261

7.1 Conditional expectation, given a discrete random variable 261
7.2 Radon–Nikodym theorem 266
7.3 Conditional expectation: The general case 271
7.4 Martingales 281
7.5 L_p spaces 291
7.6 Further limit theorems for martingales 297
7.7 Exchangeability, De Finetti's theorem 306
References 312

8. TOPICS IN THE THEORY OF STOCHASTIC PROCESSES 315

8.1 Foundations and basic concepts 315
8.2 Poisson process 321
8.3 Renewal processes 328
8.4 The Galton–Watson process; busy periods in queues 336
8.5 Markov chains 341
8.6 An invariance principle 353
References 359

Hints for Solutions of Exercises 361

Index 397