

CONTENTS

INTRODUCTION	9
Chapter 1	
BASICS OF PROBABILITY THEORY AND STATISTICS	11
1.1. Probability and random variables	11
1.2. Distribution function, density function	13
1.3. Transformation theorems	20
1.4. Statistical estimators and tests	21
1.5. Notes	25
Chapter 2	
GENERATION OF UNIFORMLY DISTRIBUTED RANDOM NUMBERS	26
2.1. Introduction	26
2.2. The concept of random sequence	28
2.3. Natural random numbers	33
2.4. Quasirandom numbers	35
2.5. Congruential generators	41
2.6. Linear recursive generators	49
2.7. The distribution of the generated numbers	55
2.8. Statistical tests	58
2.9. Notes	66
Chapter 3	
GENERAL METHODS FOR GENERATING NON-UNIFORM RANDOM NUMBERS	68
3.1. Inversion method	69
3.2. Acceptance-rejection method	74
3.3. Decomposition method	79
3.4. Comparison method	86
3.5. Economical method	91

3.6.	Ratio-of-uniforms method	96
3.7.	Variants of the acceptance-rejection method	98
3.8.	Generating discrete variables by binary search	103
3.9.	Generating discrete variables by table method	106
3.10.	Generating discrete variables by decomposition	111
3.11.	Generating discrete variables by economical method	116
3.12.	Notes	118

Chapter 4

RANDOM NUMBER GENERATORS FOR COMMON DISTRIBUTIONS

4.1.	Introduction	120
4.2.	Normal distribution	121
4.3.	Two fast normal generators	128
4.4.	Exponential distribution	139
4.5.	Gamma distribution	144
4.6.	Beta distribution	153
4.7.	Binomial distribution	156
4.8.	Poisson distribution	157
4.9.	Miscellaneous distributions	160
4.10.	Notes	164

Chapter 5

GENERATION OF RANDOM VECTORS

5.1.	General methods for generating random vectors	166
5.2.	Uniform distribution in rectangle	166
5.3.	Uniform distribution in simplex	170
5.4.	Uniform distribution in and on a hypersphere	171
5.5.	Uniform distribution in hyperellipsoid	175
5.6.	Chord method for generating points in hyperellipsoid	180
5.7.	Normal distribution	182
5.8.	Decomposition method for normal distribution	186
5.9.	Notes	191

Chapter 6

MONTE CARLO METHODS	197
6.1. Introduction	197
6.2. Crude method	198
6.3. The acceptance-rejection method	200
6.4. Error estimation	203
6.5. Comparison of methods	206
6.6. Importance sampling	209
6.7. Stratified sampling	212
6.8. Control variates method	214
6.9. Antithetic variates method	217
6.10. Multidimensional Monte Carlo methods	220
6.11. Notes	223

Chapter 7

COMPUTING THE DISTRIBUTION FUNCTION OF THE MULTIDIMENSIONAL NORMAL DISTRIBUTION	224
7.1. Formulation of the problem	224
7.2. Monte Carlo integration	226
7.3. Comparison of integration techniques	229
7.4. Importance sampling	231
7.5. Control variates method	237
7.6. Decreasing the number of variables	238
7.7. Antithetic variates technique	240
7.8. Orthonormalized estimates	241
7.9. Probability of a rectangle	243
7.10. Probability content of a line	245
7.11. A detailed orthonormalized algorithm	250
7.12. Computer experiences	252
7.13. Notes	257

Chapter 8

SIMULATION: EXAMPLES OF APPLICATION	258
8.1. Introduction	258
8.2. System of linear equations	259
8.3. Variants and applications of solving linear equations	263
8.4. A differential equation	268
8.5. Interpolation of a function of very many variables	273
8.6. Algorithms of optimization	276

8.7.	Stochastic quasi-gradient algorithms	280
8.8.	Equation of state calculations and optimization	284
8.9.	The STABIL stochastic programming model	286
8.10.	Optimal water level regulation of a lake	290
8.11.	System simulation	291
8.12.	A queuing system	293
8.13.	A SIMSCRIPT example	296
8.14.	System simulational examples	298
8.15.	Notes	302
APPENDICES		305
A1.	The Buffon's needle problem	305
A2.	Fields and polynomial rings	307
A3.	The distribution function of the χ_n^2 distribution	313
A4.	Examples with known results and exercises with unknown probabilities in evaluating the distribution function of the multidimensional normal distribution	316
A5.	Simulation related subroutines in a subroutine library	320
LIST OF ALGORITHMS		323
REFERENCES		327
SUBJECT INDEX		339