CONTENTS

Preface *page* ix

1 Theory of sets

1.1 Sets 1
1.2 Mappings 3
1.3 Cardinal numbers 5
1.4 Operations on subsets 9
1.5 Classes of subsets 14
1.6 Axiom of choice 19

2 Point set topology

2.1 Metric space 23
2.2 Completeness and compactness 29
2.3 Functions 35
2.4 Cartesian products 38
2.5 Further types of subset 41
2.6 Normed linear space 44
2.7 Cantor set 49

3 Set functions

3.1 Types of set function 51
3.2 Hahn–Jordan decompositions 61
3.3 Additive set functions on a ring 65
3.4 Length, area and volume of elementary figures 69

4 Construction and properties of measures

4.1 Extension theorem; Lebesgue measure 74
4.2 Complete measures 81
4.3 Approximation theorems 84
4.4* Geometrical properties of Lebesgue measure 88
4.5 Lebesgue–Stieltjes measure 95

5 Definitions and properties of the integral

5.1 What is an integral? 100
5.2 Simple functions; measurable functions 101
5.3 Definition of the integral 110
5.4 Properties of the integral 115
5.5 Lebesgue integral; Lebesgue–Stieltjes integral 124
5.6* Conditions for integrability 127

6 Related spaces and measures

6.1 Classes of subsets in a product space *page* 134
6.2 Product measures 138
6.3 Fubini's theorem 143
6.4 Radon–Nikodym theorem 148
6.5 Mappings of measure spaces 153
6.6* Measure in function space 157
6.7 Applications 162

7 The space of measurable functions

7.1 Point-wise convergence 166
7.2 Convergence in measure 171
7.3 Convergence in pth mean 174
7.4 Inequalities 183
7.5* Measure preserving transformations from a space to itself 187

8 Linear functionals

8.1 Dependence of $\mathscr{L}_2$ on the underlying $(\Omega, \mathscr{F}, \mu)$ 194
8.2 Orthogonal systems of functions 199
8.3 Riesz–Fischer theorem 202
8.4* Space of linear functionals 209
8.5* The space conjugate to $\mathscr{L}_p$ 215
8.6* Mean ergodic theorem 219

9 Structure of measures in special spaces

9.1 Differentiating a monotone function 224
9.2 Differentiating the indefinite integral 230
9.3 Point-wise differentiation of measures 236
9.4* The Daniell integral 241
9.5* Representation of linear functionals 250
9.6* Haar measure 254

10 What is probability?

10.1 Probability statements 261
10.2 The algebra of events 265
10.3 Probability as measure 268
10.4 Conditional probability 272
10.5 Independent trials 277

11 Random variables

11.1 Random variables as measurable functions *page* 284
11.2 Expectations 286
11.3 Distributions of random variables 290
11.4 Types of distribution function 292
11.5 Independent random variables 295
11.6 Discrete distributions 301
11.7 Continuous distributions 306
11.8 Convergence of random variables 311

12 Characteristic functions

12.1 The space of distribution functions 314
12.2 Characteristic functions 322
12.3 The inversion and continuity theorems 326
12.4 Generating functions 332

13 Independence

13.1 Sequences of independent trials 335
13.2 The Borel–Cantelli lemmas and the zero-one law 337
13.3 Sums of independent random variables 340
13.4 The central limit theorem 348
13.5 The law of the iterated logarithm 351

14 Finite collections of random variables

14.1 Joint distributions 356
14.2 Conditioning with respect to a random variable 359
14.3* Conditioning with respect to a σ-field 363
14.4 Moments 365
14.5 The multinomial distribution 368
14.6 The multinormal distribution 370

15 Stochastic processes

15.1 Renewal processes 376
15.2 The general theory of stochastic processes 380
15.3 Gaussian processes 384
15.4 Stationary processes 391

Index of notation 395

General index 397