

# Contents Volume I

<b>Notation</b>	xix
<b>Introduction</b>	1
About This Book	14
<b>Chapter 1. Necessary Results from Measure Theory</b>	16
Steinhaus' Lemma	16
Cauchy's Functional Equation	17
Slowly Oscillating Functions	18
Halasz' Lemma	21
Fourier Analysis on the Line: Plancherel's Theory	22
The Theory of Probability	24
Weak Convergence	24
Lévy's Metric	24
Characteristic Functions	27
Random Variables	29
Concentration Functions	31
Infinite Convolutions	37
Kolmogorov's Inequality 44; Lévy's Continuity Criterion 46; Purity of Type 46;	
Wiener's Continuity Criterion 48	
Infinitely Divisible Laws	49
Convergence of Infinitely Divisible Laws	53
Limit Theorems for Sums of Independent Infinitesimal Random Variables	54
Analytic Characteristic Functions	57
The Method of Moments	59
Mellin–Stieltjes Transforms	61
Distribution Functions (mod 1)	65
Quantitative Fourier Inversion	69
Berry-Esseen Theorem 74	
Concluding Remarks	76
<b>Chapter 2. Arithmetical Results, Dirichlet Series</b>	79
Selberg's Sieve Method; a Fundamental Lemma 79	
Upper Bound	84
Lower Bound	87
Distribution of Prime Numbers	89

Dirichlet Series	94
Euler Products	95
Riemann Zeta Function	96
Wiener–Ikehara Tauberian Theorem 100; Hardy–Littlewood Tauberian Theorem 102;	
Quadratic Class Number, Dirichlet’s Identity 110	
Concluding Remarks	111
<b>Chapter 3. Finite Probability Spaces</b>	115
The Model of Kubilius	119
Large Deviation Inequality 127	
A General Model	129
Multiplicative Functions	140
Concluding Remarks	144
<b>Chapter 4. The Turán–Kubilius Inequality and Its Dual</b>	147
A Principle of Duality	150
The Least Pair of Quadratic Non-Residues (mod $p$ )	153
Further Inequalities	158
More on the Duality Principle	162
The Large Sieve	165
An Application of the Large Sieve	170
Concluding Remarks	179
<b>Chapter 5. The Erdős–Wintner Theorem</b>	187
The Erdős–Wintner Theorem 187; Examples $\varphi(n)$ , $\sigma(n)$ 188; Limiting Distributions	
with Finite Mean and Variance 196	
The Function $\sigma(n)$	203
Modulus of Continuity, an Example of an Erdős Proof 207; Commentary on Erdős’	
Proof 210	
Concluding Remarks	213
Alternative Proof of the Continuity of the Limit Law	220
<b>Chapter 6. Theorems of Delange, Wirsing, and Halász</b>	225
Statement of the Main Theorems 225	
Application of Parseval’s Formula	228
Montgomery’s Lemma 229; Product Representation of Dirichlet Series (Lemma 6.6)	
230; Quantitative form of Halász’ Theorem for Mean-Value Zero 252	
Concluding Remarks	254
<b>Chapter 7. Translates of Additive and Multiplicative Functions</b>	257
Translates of Additive Functions 257; Finitely Distributed Additive Functions 258; The	
Surrealistic Continuity Theorem (Theorem 7.3) 265; Additive Functions with Finite	
First and Second Means 269	

Contents	xiii
Distribution of Multiplicative Functions	272
Criterion for Essential Vanishing 272; Modified-weak Convergence 273; Main Theorems for Multiplicative Functions 274; Examples 282	
Concluding Remarks	283
<b>Chapter 8. Distribution of Additive Functions (mod 1)</b>	284
Existence of Limiting Distributions 284; Erdős' Conjecture 285	
The Nature of the Limit Law	291
The Application of Schnirelmann Density 293; Falsity of Erdős' Conjecture 302; Translation of Additive Functions (mod 1), Existence of Limiting Distribution 302	
Concluding Remarks	305
<b>Chapter 9. Mean Values of Multiplicative Functions, Halász' Method</b>	308
Halász' Main Theorem (Theorem (9.1) ) 308; Halász' Lemma (Lemma (9.4) ) 311; Connections with the Large Sieve 317; Halász's Second Lemma (Lemma (9.5) ) 318; Quantitative Form of Perron's Theorem (Lemma (9.6) ) 322; Proof of Theorem (9.1) 326; Remarks 330	
<b>Chapter 10. Multiplicative Functions with First and Second Means</b>	333
Statement of the Main Result (Theorem 10.1) 333; Outline of the Argument 335	
Application of the Dual of the Turán–Kubilius Inequality	335
Study of Dirichlet Series	337
Removal of the Condition $p > p_0$	340
Application of a Method of Halász 343	
Application of the Hardy–Littlewood Tauberian Theorem	348
Application of a Theorem of Halász	351
Conclusion of Proof	354
Concluding Remarks	356
<b>References (Roman)</b>	I
<b>References (Cyrillic)</b>	XIX
<b>Author Index</b>	XXIII
<b>Subject Index</b>	XXIX