

Contents

Preface vii

Preface to the first edition ix

1 | Distribution function

- 1.1 Monotone functions 1
- 1.2 Distribution functions 7
- 1.3 Absolutely continuous and singular distributions 10

2 | Measure theory

- 2.1 Classes of sets 15
- 2.2 Probability measures and their distribution functions 20

3 | Random variable. Expectation. Independence

- 3.1 General definitions 32
- 3.2 Properties of mathematical expectation 39
- 3.3 Independence 49

4 | Convergence concepts

- 4.1 Various modes of convergence 64
- 4.2 Almost sure convergence; Borel–Cantelli lemma 71
- 4.3 Vague convergence 79

- 4.4 Continuation 86
 4.5 Uniform integrability; convergence of moments 94

5 | Law of large numbers. Random series

- 5.1 Simple limit theorems 101
 5.2 Weak law of large numbers 107
 5.3 Convergence of series 115
 5.4 Strong law of large numbers 123
 5.5 Applications 131
 Bibliographical note 141

6 | Characteristic function

- 6.1 General properties; convolutions 142
 6.2 Uniqueness and inversion 152
 6.3 Convergence theorems 160
 6.4 Simple applications 166
 6.5 Representation theorems 178
 6.6 Multidimensional case; Laplace transforms 187
 Bibliographical note 195

7 | Central limit theorem and its ramifications

- 7.1 Liapounov's theorem 196
 7.2 Lindeberg–Feller theorem 205
 7.3 Ramifications of the central limit theorem 214
 7.4 Error estimation 224
 7.5 Law of the iterated logarithm 231
 7.6 Infinite divisibility 238
 Bibliographical note 249

8 | Random walk

- 8.1 Zero-or-one laws 250
 8.2 Basic notions 257
 8.3 Recurrence 266
 8.4 Fine structure 275
 8.5 Continuation 285
 Bibliographical note 293

9 | Conditioning. Markov property. Martingale

- 9.1 Basic properties of conditional expectation 295
 9.2 Conditional independence; Markov property 306
 9.3 Basic properties of smartingales 318
 9.4 Inequalities and convergence 330
 9.5 Applications 343
 Bibliographical note 356

General Bibliography 358